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ORIGINAL PAPER

Optimizing of Multi-objective Inventory Model by Different Fuzzy Techniques

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Abstract
A multi-objective, multi item inventory model is constructed for deteriorating items where the demand is considered as exponential time function under limited storage space as well as budget. By using Fuzzy non linear programming (FNLP) and Intutionistic fuzz optimization (IFO) techniques results are obtained and then compared. The objective of this work is to use FNLP and IFO techniques for multi-objective inventory model and to compare these techniques through numerical results. The major goal of the paper is to find optimal quantity to be replenished and identify time point when shortages will occur. In this paper FNLP and IFO are applied to multi item multi-objective inventory model with budget and warehouse space constraint and investigating for multi-objective inventory model which method either FNLP or IFO gives efficient solution. In case of maximization objective IFO works well than FNLP while in case of minimization FNLP works better. By observing objectives, the above methods can apply to various inventory problems. All these results along with relation of profit and shortage cost with budget, warehouse space is studied through sensitivity analysis. The result shows that the IFO better results for maximizing profit while FNLP works better in case of minimizing shortage cost.

Keywords Fuzzy non-linear programming (FNLP) · Intuitionistic fuzzy optimization (IFO) · Multi-objective inventory model

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Introduction

An inventory control policy deals with the nature of relative parameters such as deterioration rate, demand, holding cost, shortage cost etc. Demand is the key factor of inventory management. So many researchers have used different types of demands such as constant, price dependent, stock dependent, time dependent, exponential, ramp type etc. In classical inventory models generally demand is assumed to be constant but in real life situations it occurs rarely. Exponential increasing demand is used by Mehta and Shah [24]. They have developed a lot-size inventory model with exponentially increasing demand by allowing complete backlogging for deteriorating items. Chang and Dye [9] and Lee and Hsu [19] constructed inventory model for general time dependent demand in there paper they used exponentially increasing demand in example for illustration purpose. Sarkar and Chakrabarti [33] developed production inventory model with exponentially increasing demand. Also, Yadav and Swami [39] and Shukla and Chandel [35] used similar demand. Various inventory models have been developed by various situations such as Wee et al. [38] proposed new approach of finding solution condition is analysed using the production time and the time to eliminate backorders as decision variables. Cárdenas-Barrón et al. [3] discussed a brief introduction to the papers included in the special issue “Celebrating a century of the economic order quantity model in honour of Ford Whitman Harris” published by the International Journal of Production Economics. Shah et al. [34] developed inventory model for trended demand with fixed credit period to the retailer and customer. Kumar and Sana [18] deals with a classical Economic Order Quantity model with backlogging and the demand of end customers is dependent on promotional effort and selling price simultaneously. Salvietti et al. [31] discussed a stochastic version of the economic lot sizing problem with pricing. Cárdenas-Barrón et al. [5] proposed an alternative heuristic algorithm for EPQ model with Just in Time philosophy and a budget constraint. Chakrabarty et al. [6] develops a production-inventory model of a single product with imperfect production process in which inflation and time value of money are considered under shortages. Geetha and Udayakumar [15] deals with an economic order quantity model for non-instantaneous deteriorating items with price and advertisement dependent demand pattern and the salvage value for deteriorating items is considered in this model. Chowdhury et al. [10] develop an inventory model for finding the strategy for a firm that sells a seasonal item over a finite planning time. Mishra [25] studied an inventory model with Weibull deterioration and time dependent demand with partial backlogging. In the same year Mishra [26] again studied an EOQ model with controllable deterioration rates under shortages. Li et al. [20] developed inventory model by taking into account advance-cash-credit payment scheme as well as effect of expiration date on demand of perishable products. Feng et al. [14] built an inventory model with demand considered to be a function of price, freshness, and displayed stocks. He also demonstrated that the total profit is strictly pseudo-concave with a unique local maximum solution. Mishra [27] developed an inventory model with stochastic deterioration rate with stock and price dependent demand rate. Tiwari et al. [36] discussed set up relation between EOQ and permissible delay in payment as well as shortages. Cárdenas-Barrón et al. [4] discussed two EPQ inventory models have been developed with and without shortages and nonlinear stock dependent demand and nonlinear holding cost where the supplier offers a trade credit period to the retailer. Mishra [28] studied a deteriorating inventory model with three-rates-of-production rate under selling price and advertising cost. San-José et al. [32] developed deterministic inventory model by assuming demand rate is additive function of selling price and a time-power function. He also considered that the holding cost is a power function of the amount of time.

In a real life inventory problem encounters more than one objectives like maximizes profit, minimizing wastage cost and minimizing shortage cost or total inventory cost are involved to achieve them simultaneously. Such problems are modelled as multiple objective decision making problems. Most of times these objectives are conflict with each other. Generally, the DM selects a compromise solution from a set of possible solutions. So many methods like assigning priorities to the objectives by Mahapatra and Maiti [22], IFO by Faritha et al. [12], Anglov [1] etc., exist for finding compromise solutions. Mahapatra and Roy [21], Kar et al. [17], Nayebi [29] and Faritha et al. [13] are used fuzzy nonlinear programming technique to obtain compromise solution for multi objective inventory problem.

IFO technique is another interesting technique is used to get compromise solution of multi-objective problem. Angelov [1] implemented intuitionistic fuzzy optimization in linear programming. Banerjee and Roy [2] and Chakrabortty [7] used IFO technique for inventory problem. Multi item multi-objective inventory model is proposed with available space and limited budget constraints. The research questions of our model are: (a) Is the budget affects the profit significantly? (b) How warehouse space affects profit? (c) How budget affects the shortage cost? and (d) Is there any relationship between warehouse space and shortage cost?

To solve multi-objective inventory problem two methodologies used which one suitable for getting optimum result for specific objective? FNLP method uses only membership function while IFO uses both membership and non-membership function so comparing both methods for multi objective inventory problem. A multi-objective multi item inventory model is constructed for deteriorating items. Demand is considered as exponential time function. Available space and budget are limited. By using FNLP and techniques results are obtained and compared. The main objective is to find the optimal replenishment strategies which maximize the total profit. Table 1 presents a motivation and contribution with comparison of some research works related to this study.

Inventory models on relevant ordering policies reported in the extant literature are considered, mapped and illustrated in Table 1. It is evident from Table 1, that there is no research work done by any researcher multi objective inventory model by FNLP and IFO techniques. The rest part of this paper are organised are follows. The notations of the model and assumptions are introduced in "Model and Assumptions" section. In "Mathematical Analysis" section, a mathematical model is established with objectives maximizing profit andminimizing the shortage cost for limited budget and available warehouse space constraints. In "Methodology" section, solution procedure of proposed inventory model is discussed. Numerical example is given to demonstrate the proposed model in "Numerical Example" section. Sensitivity analysis of the optimal solution with respect to limited budget and available warehouse space is carried out in "Sensitivity Analysis" section. The results from "Sensitivity Analysis" section and overall conclusions are discussed in "Conclusion" section. The article ends with scope of future research.

Model and Assumptions

Notations

C_i:	Purchasing cost per unit of ith item
P_i:	Selling price per unit of ith item
Q_i:	Initial stock level of ith item
θ_i:	Deterioration rate of ith item

Table 1 Author's contribution table

Reference	Objective	Constraints	Contributions	Limitations
Kar et al. [17]	Maximizes profit and the wastage cost as well as the total production cost is minimized	Multi-objective inventory model with production rate is function of unit cost	Profit, wastage cost, total production cost, determined by FNLP and fuzzy goal programming techniques	Production rate is finite, Model is constructed without shortage
Wee et al. [37]	Maximizing profit as well as return on inventory investment	Fuzzy demand is used with shortage cost constraint	Proposed inverse weight FNLP and the conventional fuzzy additive goal programming are compared	Replenishment is instantaneous
Prasath and Seshaiah [30]	Minimize the total expenditure of the organization by reducing the number of warehouses allocating to the selling stores	Model without shortage with budget and warehouse space limitation	By using numerical example explained	Replenishment is instantaneous No back-order is allowed.Lead time is zero
Banerjee and Roy [2]	Two models had been formed for uniform and exponential lead-time demand then the results were compared using FNLP and IFO	Model considered several fuzzy costs	A minimized value of Expected Annual Cost for different data for each demand rate is determined	Comparison is done using numerically
Faritha and Henry Amirtharaj [12]	Used Ranking function method to convert fuzzy inventory model into crisp model. Then crisp model is solved using FNLP	Multi objective, multi item inventory model under the constraint on available space and limited investment in fuzzy environment	Using FNLP optimum results obtained for maximizing profit, minimizing Total and wastage cost	Model is constructed without shortage
Mahapatra and Maiti [22]	Profit for n different items maximized simultaneously using FNLP and FAGP for stock dependent demand Production inventory model	Constraint on available space and limited investment	Optimal solutions obtained and numerically FAGP is better than FNLP	Comparison is done using numerically

Table 1 continued

Reference	Objective	Constraints	Contributions	Limitations
Zimmeraman [40]	Application of fuzzy linear programming approaches to the linear vector maximum problem	Theoretical paper of fuzzy linear programming and not deals with inventory model, so constraints and limitations are not applicable	FNLP has always efficient solutions	Theoretical paper of fuzzy linear programming and not deals with inventory model, so constraints and limitations are not applicable
Faritha and Henry Amirtharaj [13]	Solved multi objective, multi item inventory model. Demand is taken to be stock dependent demand. Used Ranking function method to convert fuzzy inventory model into crisp model. Then crisp model is solved using IFO	Method is illustrated through example	IFO technique used for getting optimum results	Model is constructed without shortage
Mandal et al. [23]	To maximize profit for inventory model formulated in fuzzy environment having stock-dependent demand	Available space is limited. Selling price as well as holding cost are dependent on purchasing price	Model is solved by FNLP method and results were obtained. Parametric study for some parameters were presented	Comparison is done using numerically
Jadhav and Bajaj [16]	Minimizing the total average cost as well as wastage cost and simultaneously maximize profit	Multi-objective inventory model having stock-dependent demand is formulated in crisp and fuzzy environment with constraint on available space and limited investment	The problem is solved by different methods such as FNLP, Weighted Fuzzy Programming Technique,Weighted Goal Programming etc.	Comparisons of methods is not done
Durga et al. [11]	Presented a method based on ranking function which convert a fuzzy multi objective non programming to crisp	Objectives non linear and constraints are linear	Crisp problem is solved using FNLP	Objectives and constraints are nonlinear case not discussed

Table 1 continued

Reference	Objective	Constraints	Contributions	Limitations
Chakrabortty et al. [7]	A manufacturing inventory model with shortages is formulated. Carrying cost, shortage cost, setup cost and demand are represented fuzzy number	Demand is of imprecise nature i.e., demand is fuzzy	Problem has been solved by intuitionistic fuzzy programming technique. The proposed method is illustrated with a numerical example and Pareto optimality test has been applied as well	Non linear membership function not used
Chakrabortty et al. [8]	Manufacturing inventory models with shortages where carrying cost, shortage cost, setup cost and demand quantity are considered as fuzzy number	Demand is of imprecise nature i.e., demand is fuzzy	By using IFO technique optimal solution is derived. Solutions obtained satisfies pareto optimality test	Non linear membership function not used
This paper	Multi-item Multi objective inventory model for profit maximization and minimizing shortage cost with exponential demand is formulated	Constraint on available space and limited investment	Model has been solved using FNLP and IFO. Comparison between FNLP and IFO technique. Sensitivity analysis is done with respect to Warehouse space availability and limited budget are presented	Deterioration rate is considered to be constant. Comparison is done using numerically. On-linear membership function not used

$D(t) = -a_i e^{b_i t_i}$:	Demand rate of per unit of ith item
$Q_i(t)$:	Inventory level at time t of ith item
C_{hi}:	Holding cost per unit of ith item
C_{di}:	Deteriorating cost per unit of ith item
C_{2i}:	Shortage cost per unit of ith item
W_i:	Warehouse space required for ith item
T:	Time period for each cycle
B:	Capital investment available for purchasing all items
W:	Warehouse space available to store items

Assumptions

- Replenishment is instantaneous.
- Shortages are allowed.
- Selling price is known and constant.

Mathematical Analysis

From Fig. 1, the initial stock level is Q_i of ith item at time $t = 0$ i.e., $Q_i(0) = Q_i$ then it decreases due to demand mainly and partially by deterioration. The stock of ith item reaches to zero level at $t = t_i$, then shortages occurs.

The differential equation explaining the state of inventory in the interval $(0, t_i)$ is given by;

$$\frac{dQ_i(t)}{dt} + \theta_i Q_i(t) = -a_i e^{b_i t_i}, \quad 0 \le t \le t_i \tag{1}$$

Solving above differential equation using boundary condition. $Q_i(0) = Q_i$ we get

$$Q_i(t) = -a_i \left[\frac{e^{b_i t}}{b_i + \theta_i}\right] + \left(Q_i + \frac{a_i}{b_i + \theta_i}\right) e^{-\theta_i t}, \quad 0 \le t \le t_i \tag{2}$$

using boundary condition $Q_i(t_i) = 0$ we get;

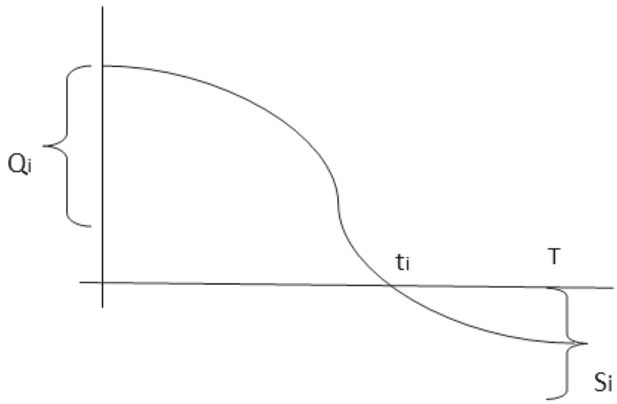

Fig. 1 Inventory model

$$- a_i\left[\frac{e^{b_i t_i}}{b_i+\theta_i}\right]+\left(Q_i+\frac{a_i}{b_i+\theta_i}\right)e^{-\theta_i t_i}=0$$
$$\Rightarrow t_i=\left[\frac{1}{\theta_i+b_i}\right]\ln\left(1+\frac{Q_i(b_i+\theta_i)}{a_i}\right)$$

The differential equation explaining the state of inventory in the interval at (t_i, T) is given by;

$$\frac{dQ_i(t)}{dt}=-a_i e^{b_i t_i},\quad t_i\le t\le T \tag{3}$$

Integrating both sides and solving using condition $Q_i(t_i)=0$ we get

$$Q_i(t)=-\left(\frac{a_i e^{b_i t}}{b_i}\right)+\left(\frac{a_i e^{b_i t_i}}{b_i}\right),\quad t_i\le t\le T$$

Total Holding cost over the time period $(0, t_i)$ is given by; $c_{hi}\int_0^{t_i} Q_i(t)dt = c_{hi}\left(\left(\frac{a_i}{b_i*(b_i+\theta_i)}\right)\left(1-e^{b_i t_i}\right)+\left(Q_i+\frac{a_i}{(b_i+\theta)_i}\right)\left(\frac{1}{\theta_i}\right)\left(1-e^{\theta_i t_i}\right)\right)$.

Total shortage cost is given by;

$$-c_{2i}\int_{t_i}^{T_i} Q_i(t)dt=-c_{2i}\left(-\frac{a_i}{b_i^2}\left(e^{b_i t_i}-e^{b_i T}\right)+\frac{a_i}{b_i}(T-t_i)e^{b_i t_i}\right)$$

Then the total profit is given by;

$$PF=\sum_{i=1}^{n}\left((p_i-c_i)Q_i-(c_{hi}+c_{di}\theta)\int_0^{t_i}Q_i(t)dt\right)$$
$$PF=\sum_{i=1}^{n}\left((p_i-c_i)Q_i-(c_{hi}+c_{di}\theta)\left(\left(\frac{a_i}{b_i(b_i+\theta)_i}\right)\left(1-e^{b_i t_i}\right)+\left(Q_i+\frac{a_i}{(b_i+\theta)_i}\right)\right.\right.$$
$$\left.\left.\times\left(\frac{1}{\theta_i}\right)\left(1-e^{\theta_i t_i}\right)\right)\right)$$
$$SC=\sum_{i=1}^{n}\left(c_{2i}\left(-\frac{a_i}{b_i^2}\left(e^{b_i t_i}-e^{b_i T_i}\right)+\frac{a_i}{b_i}(T_i-t_i)e^{b_i t_i}\right)\right)$$

Hence the inventory problem is maximizing profit as well as minimizing shortage cost subject to limitations on capital investment and storage area, i.e.,

$$\begin{aligned}&Max\ \ PF\\&Min\ \ SC\\&Subject\ to\\&\sum_{i=1}^{n}w_i Q_i\le W\\&\sum_{i=1}^{n}c_i Q_i\le B\end{aligned}$$

$$t_i = \left[\frac{1}{\theta_i + b_i}\right] \ln\left(1 + \frac{Q_i(b_i + \theta_i)}{a_i}\right)$$
$$Q_i \geq 0, \quad i = 1, 2, \ldots, n \tag{4}$$

Methodology

Fuzzy Nonlinear Programming Technique FNLP

This method is given by Zimmerman [40]. This algorithm has following steps.

Step 1: Solve the multi-objective problem as a single objective non-linear programming problem by considering one objective ignoring the others

Step 2: From the result of Step 1, determine the solutions for every objective at each solution obtained. With these values of all objectives at solution, find lower bound L_r and upper bounds U_r for rth objective functions

Step 3: Define the linear membership function $\mu_r(X)$, corresponding to the rth objective

$$\mu_r(Z_r) = \begin{cases} 1 & Z_r < L_r \\ \frac{U_r - Z_r}{U_r - L_r} & L_r \leq Z_r \leq U_r \\ 0 & Z_r > U_r \end{cases}$$

Step 4: According to Zimmermann [41], the equivalent crisp non-linear programming problem for the multi-objective problem is

$$\begin{aligned} & Max \ \ \alpha \\ & Subject \ to \\ & \frac{U_r - Z_r}{U_r - L_r} \geq \alpha, \quad r = 1, 2 \\ & \sum_{i=1}^{n} w_i * Q_i \leq W \\ & \sum_{i=1}^{n} c_i * Q_i \leq B \\ & Q_i \geq 0, \quad i = 1, 2, \ldots, n \end{aligned}$$

Solve above problem which gives solution original multi-objective problem.

Step 5: Stop.

By following above method crisp multi-objective inventory model given in (4) become;

$$\begin{aligned} & Max = \alpha \\ & Such \ that \\ & \left(\frac{U_1 - SC}{U_1 - L_1}\right) \geq \alpha \\ & \left(\frac{PF - L_2}{U_2 - L_2}\right) \geq \alpha \\ & \sum_{i=1}^{n} w_i * Q_i \leq W \end{aligned}$$

$$\sum_{i=1}^{n} c_i * Q_i \leq B$$
$$Q_i \geq 0, \quad i = 1, 2, \ldots, n$$
$$t_i = \left[\frac{1}{\theta_i + b_i}\right] * \ln\left(1 + \frac{Q_i * (b_i + \theta_i)}{a_i}\right)$$

Intuitionistic Fuzzy Optimization IFO

This technique is introducing by [1]. This algorithm has following steps.

Step 1: Solve the multi-objective problem as a single objective non-linear programming problem by considering one objective ignoring the others

Step 2: From the result of Step 1, determine the solutions for every objective at each solution obtained. With these values of all objectives at solution, find lower bound L_r and upper bounds U_r (Suppose we have two objectives with some constraint then we solve multi-objective problem as single objective and obtain two solutions for each objective then by using solution obtained for first objective in second objective we get another value for second objective from these values min value is L_2 and max value is U_2) for rth objective functions

Step 3: Define the membership function $\mu_r(X)$ as well as non-membership function $v_r(X)$ corresponding to the rth objective lower bounds and upper bounds for non-membership functions are given by L^r and U^r (In case of minimization problem, the lower bound for non-membership function(L^r) is always greater than that of the membership function(L_r)this lemma is given in Banerjee and Roy [2]. For objective function of maximization problem, the upper bound of non-membership function (U^r) is always less that the upper bound of membership function (U_r) proved by Mahapatra and Maiti, [22]. Write all equation

$$\mu_r(Z_r) = \begin{cases} 1 & Z_r < L_r \\ \frac{U_r - Z_r}{U_r - L_r} & L_r \leq Z_r \leq U_r \\ 0 & Z_r > U_r \end{cases}, \quad v_r(Z_r) = \begin{cases} 1 & Z_r < U^r \\ \frac{Z_r - L^r}{U^r - L^r} & L^r \leq Z_r \leq U^r \\ 0 & Z_r < L^r \end{cases}.$$

Step 4: Following Angelov together with linear membership function the equivalent crisp non-linear programming problem for the multi-objective problem is;

$$Max \;\; \alpha - \beta$$
$$Subject\ to$$
$$\frac{U_r - Z_r}{U_r - L_r} \geq \alpha, \quad (r = 1, 2)$$
$$\frac{Z_r - L^r}{U^r - L^r} \leq \beta, \quad (r = 1, 2)$$
$$\sum_{i=1}^{n} w_i Q_i \leq W$$
$$\sum_{i=1}^{n} c_i Q_i \leq B$$

$$Q_i \geq 0, \quad i = 1, 2, \ldots$$
$$\beta \geq 0, \quad \alpha > \beta, \quad \alpha + \beta < 1$$

Solve above problem which gives solution original multi-objective problem.

Step 5: Stop.

Following the above method model given in Eq. (4) reduced to

$$
\begin{aligned}
&Max \;\; \alpha - \beta \\
&Subject\ to \\
&\frac{U_r - Z_r}{U_r - L_r} \geq \alpha, \quad (r = 1, 2) \\
&\frac{Z_r - L^r}{U^r - L^r} \leq \beta, \quad (r = 1, 2) \\
&\sum_{i=1}^{n} w_i Q_i \leq W \\
&\sum_{i=1}^{n} c_i Q_i \leq B \\
&t_i = \left[\frac{1}{\theta_i + b_i}\right] \ln\left(1 + \frac{Q_i(b_i + \theta_i)}{a_i}\right) \\
&Q_i \geq 0, \quad i = 1, 2, \ldots \\
&\beta \geq 0, \quad \alpha > \beta, \quad \alpha + \beta < 1
\end{aligned}
$$

Numerical Example

Input

Let, $P_1 = P_2 = \$10$, $C_1 = \$7$, $C_2 = \$6.75$, $C_{h1} = C_{h2} = \$2.2$, $C_{d1} = C_{d2} = \$4$, $\theta_1 = 0.05$, $\theta_1 = 0.06$, $a_1 = 100$, $a_2 = 50$, $b_1 = 0.25$, $b_2 = 0.5 w_1 = 2\,\text{Sq.ft}$, $w_2 = 2.2\,\text{Sq.ft}$, $w = 200\,\text{Sq.ft}$, $B = \$650$, $C_{21} = C_{22} = \$1$.

Using LINGO Software, we obtained the following results.

Output

(a) FNLP:

$$\alpha = 0.7618, \; Q_1 = 58.15965, \; Q_2 = 35.98258, \; t_1 = 0.5360\,\text{year},$$
$$t_2 = 0.604672\,\text{year}, \; PF = \$225.6571, \; SC = \$18.4467.$$

(b) IFO:

$$\alpha = 0.76183, \; \beta = 0.17957, \; Q_1 = 58.15965, \; Q_2 = 35.98258, \; t_1 = 0.536082\,\text{year},$$
$$t_2 = 0.604672\,\text{year}, \; PF = \$225.6571, \; SC = \$18.4467.$$

Sensitivity Analysis

Sensitivity analysis gives effect of different parameters on output. Here effect of capital investment (B), Available warehouse space (W) is checked through Tables 1 and 2.

From, Sensitivity Analysis Tables 1 and 2, the following observation and managerial application can be made.

- As capital investment and available warehouse space increases the parameters such as PF, t_1, t_2, Q_1, Q_2 increases and SC decreases. Satisfaction level α and non satisfaction level β of decision maker changes with change in capital investment. So, to reduce shortage cost and maximize profit decision maker has to increase capital investment and available warehouse.

Conclusion

In this study a multi-objective and multi item inventory model is formulated with exponential demand under limited available warehouse space and budget. Here, the results are compared numerically both in FNLP and IFO techniques. The objective of this work is to use FNLP and IFO techniques for multi-objective inventory model and to compare these techniques through results. From Tables 2 and 3, it is observed that both capital investment and warehouse space both are positively related with profit and negative correlated with shortage cost. It is alsoobserving that IFO shows better results for maximizing profit while FNLP works better in case of minimizing shortage cost. Various types of demands can be used and effectiveness of FNLP and IFO can be investigated. Deterioration rate can be modelled using Weibull, Pareto distribution. In this paper all parameters are considered to be crisp but they can be considered in stochastic or fuzzy environments to develop inventory model. Also, this model can be extended by assuming different constraints, like limitation of number of orders, utilization of whole space etc. and different demands, like Verhulst (Verhulst demand came from a mathematical model that determines the rate of increase in the population of species in a given ecological environment in 1840 its interpretation for retailer point view is “the more population, the more demand for the product”) demand, income dependent demand, income

Table 2 Effect of capital investment

Capital investment	α	β	t_1	t_2	Q_1	Q_2	PF	SC
500								
IFO	0.70401	0.0405	0.41327	0.49387	43.9984	28.4461	184.58	28.9899
FNLP	0.75394		0.41518	0.49048	44.214	28.2225	184.552	28.9764
550								
IFO	0.6059	0.3940	0.4357	0.5646	46.551	33.2058	199.081	25.38
FNLP	0.77790		0.45235	0.5357	48.4486	31.2384	198.977	25.20
600								
IFO	0.74118	0.1905	0.49619	0.56754	53.5024	33.4049	212.626	21.6725
FNLP	0.74662		0.49642	0.56715	53.5288	33.3776	212.623	21.671
650								
IFO	0.76183	0.1796	0.53608	0.60467	58.1597	35.9826	225.657	18.4467
FNLP	0.7618		0.53608	0.60467	58.1597	35.9826	225.657	18.4467

Table 3 Effect of available warehouse space

Warehouse space	α	β	t_1	t_2	Q_1	Q_2	PF	SC
100								
IFO	0.65347	0.17111	0.31385	0.28646	32.9103	15.5361	131.158	43.58465
FNLP	0.71798		0.3155	0.28366	33.091	15.3718	131.144	43.5782
150								
IFO	0.71571	0.19325	0.44995	0.43123	48.1734	24.3878	184.156	28.78843
FNLP	0.72939		0.45031	0.43065	48.2141	24.3508	184.152	28.78706
180								
IFO	0.67003	0.18107	0.52562	0.5182	56.9325	30.0614	212.256	21.5163
FNLP	0.72138		0.52696	0.51606	57.0902	29.918	212.244	21.51065
200								
IFO	0.76184	0.17956	0.53608	0.60467	58.1597	35.9826	225.657	18.44669
FNLP	0.7618		0.53608	0.60467	58.1597	35.9826	225.657	18.4467

and price dependent demand etc. comparison of FNLP and IFO can be done. Non linear membership function can be used in FNLP and IFO. By considering various situations crisp inventory models can be obtained and by applying FNLP and IFO, there efficiency can be investigated.

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