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Unit - II ① [2023-24]

1 Conservation Theorems [2023-24]

Dynamics: The science of the motion of bodies and the action of forces in producing or changing their motion.

* **Newtons Sec. law** → Law of conservation of linear momentum e.g. for atomic and nuclear physics.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

* The law of conservation of angular momentum.

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$ ⟶ called Newton's law for the rotational motion.

* When conservative forces are acting on a particle or a sys. of particles, then:

$E = K.E. + P.E.$ --- (total mechanical energy is conserved)

* When non-conservative forces → (e.g. friction)

Energy $\xrightarrow{\text{conversion}}$ Heat, light, Sound.

* Conservation Principle in dynamics:

→ Conservation of linear momentum

— " — of Angular momentum

— " — of Energy

Basic Formulae:

1. $F = ma$, $a = \frac{dv}{dt} = \frac{d^2 r}{dt^2}$

2. $F = ma = \frac{dP}{dt} = \frac{d}{dt}(mv)$ $\because P = mv$

3. Torque $(\vec{\tau}) = \vec{r} \times \vec{F}$

4. Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

5. Work done = Force × displacement.
$$= \vec{f} \cdot \vec{dr} \text{ or } \vec{f} \cdot \vec{ds}$$

6. For conservative force,
$$\Delta T + \Delta U = 0$$
$$\therefore W = \Delta T = -\Delta U$$

$\therefore$ Conservative force $\Rightarrow$ −ve Gradiant of the potential.

i.e. $f = -\nabla U = -\frac{dU}{dr}$ $\because \nabla = \frac{d}{dr}$

1.2 Single Particle

1.2 (a) : Conservation of Linear Momentum

Thm: The linear momentum of a particle remains constant when external force acting on it is zero

Proof:

Consider a particle of mass m moving with velocity $\vec{v}$ due to force $\vec{F}$ acting on it.

$\therefore$ linear momentum $(\vec{p}) = m\vec{v}$

Differentiating w.r.t. "t"

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) \quad \text{——} \quad (1)$$

According to Newton's law of motion

$$F = ma$$

$$F = m\frac{dv}{dt}$$

$$F = \frac{d}{dt}(mv) \quad \text{——} \quad (2)$$

$\therefore$ Equating eqn (1) & (2)

$$F = \frac{dp}{dt}$$

When $F = 0$, then $\frac{dp}{dt} = 0$

$$\Rightarrow \quad \vec{p} = \text{constant.}$$

Thus, the linear momentum of a particle is conserved when the external force F acting on it is zero.

b] Conservation of Angular Momentum:-

Thm: The angular momentum of a particle remains constant when external torque acting on it is zero.

Proof:

Let a particle of mass "m" is moving along a circular path of radius $\vec{r}$ with linear velocity $\vec{v}$

$\therefore$ Linear momentum $\vec{P} = m\vec{v}$

And angular momentum $\vec{L} = \vec{r} \times \vec{P}$ ——— (1)

The torque responsible for motion of the particle along curve ~~is~~ of radius r is

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{——— (2)}$$

Where, F = force

r = ⊥ar dist. betⁿ two forces forming couple

~~We know~~,

According to Newtons Sec Law

$$F = \frac{dP}{dt}$$

$\therefore$ Eqⁿ (2) becomes.

$$\vec{\tau} = r \times \frac{dP}{dt} \quad \text{——— (3)}$$

~~Consider the term~~, differentiating. eqⁿ (1) w.r.t. "t"

$$\frac{dL}{dt} = \frac{dr}{dt} \times P + r \times \frac{dP}{dt}$$

$\because P = mV$

$$\frac{dL}{dt} = V \times (mV) + r \times \frac{dP}{dt}$$

$$\frac{dL}{dt} = r \times \frac{dP}{dt} \quad \text{——— (3p)}$$

$\therefore m(V \times V) = 0$

From Eqn 3 & 4

$$\tau = \frac{dL}{dt}$$

If there is no torque acting on the particle i.e. $\tau = 0$ then $\frac{dL}{dt} = 0$

$$\Rightarrow L = \text{constant}$$

Thus, when external torque τ acting on the particle is zero then its angular momentum L remains constant. i.e. it is conserved.

1.2 c] Work Energy Thm.

Thm: The workdone by or on the particle is equal to the change in K.E. of the particle.

Proof:

If force $\vec{F}$ is acting on the particle, due to this force particle gets displaced from position 1 to 2.

$\therefore$ Workdone $W_{12} = \int_1^2 F.ds$

Where, ds = displacement of the particle along the path.

We know, $F = ma = m.\frac{dv}{dt}$ and $v = \frac{ds}{dt}$

$\Rightarrow ds = v\,dt$

$$\therefore W_{12} = \int_1^2 m.\frac{dv}{dt}.v\,dt$$

$$W_{12} = \int_1^2 m\,v\,dv$$

$$= m\left[\frac{v^2}{2}\right]_{v_1}^{v_2}$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = T_2 - T_1$$

Where, $T_2 = \frac{1}{2}mv_2^2$ = K.E. of particle in position 2

$T_1 = \frac{1}{2}mv_1^2$ = K.E. of particle in position 1

Thus, the Work Energy Thm. is proved.

1) When $T_2 > T_1$, then W_{12} is positive $\Rightarrow$ Work is done on particle

2) When $T_2 < T_1$, then W_{12} is negative $\Rightarrow$ Work is done by the particle.

1.2 d] Conservation of Energy of a Particle (i.e. Single particle):

Conservative force

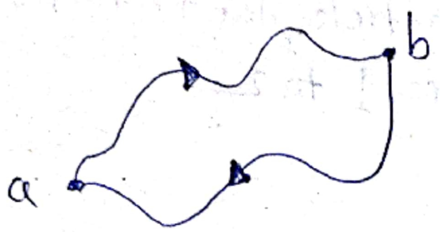


fig: Motion of a particle in a round trip

1. If work done by the forces in one round trip (a→b→a) is zero

e.g. Spring force, Gravitational force

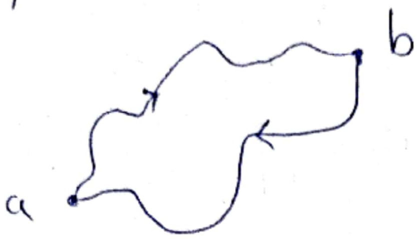
Non-Conservative force

1. If work-done in one round trip is non-zero

e.g. friction, induction etc.

Thm: In conservative force field the total energy of a particle in initial position is equal to the total energy of a particle in final position.

Proof:



If $\vec{f}$ is a conservative force acting on a body, then work done in moving it from one position to another say $1 \to 2$ is

$$W_{12} = \int_1^2 f \cdot dr = \int_1^2 f\, dr$$

$\because f \cdot dr = f\,dr \cos\theta$
$\because \vec{dr}$ is along $\vec{f}$
$\therefore \theta = 0 \Rightarrow \cos\theta = 1$

But $f = ma = m\dfrac{dv}{dt} = m\dfrac{dv}{dr}\overset{v}{\dfrac{dr}{dt}} = mv\dfrac{dv}{dr}$

$$\therefore W = \int_1^2 mv\frac{dv}{dr}\,dr = \int_1^2 mv\,dv = m\int_1^2 v\,dv = m\left[\frac{v^2}{2}\right]_1^2$$

$$= m\frac{v_2^2}{2} - m\frac{v_1^2}{2}$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W_{12} = T_2 - T_1 \qquad (1)$$

The conservative force can also be defined as the $-ve$ gradient of the potential.

$$f = -\nabla U = -\frac{dU}{dr}$$

$\because \Delta T + \Delta U = 0$
$W = \Delta T = -\Delta U$
$W = -\nabla\Delta U$
$\Delta U = -W$

$$W_{12} = \int_1^2 F \cdot dr = \int_1^2 F\,dr$$

{ ∵ $F \cdot dr = F\,dr\cos\theta$
$F \cdot dr = F\,dr$
∵ $\theta = 0 \Rightarrow \cos\theta = 1$ }

$$W_{12} = \int_1^2 -\frac{dU}{dr}\,dr$$

$$W_{12} = -\int_1^2 dU = -[U_2 - U_1]$$

$$W_{12} = U_1 - U_2 \quad \text{——— (2)}$$

From eqn (1) and (2) we get.

$$T_2 - T_1 = U_1 - U_2$$

$$\Rightarrow \quad T_1 + U_1 = T_2 + U_2 = \text{constant}$$

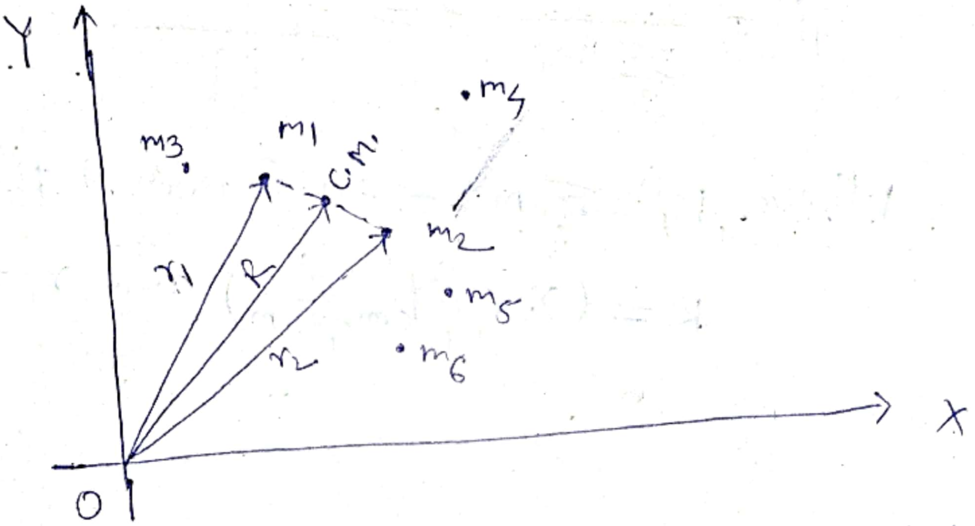
Hence, $T + U$ = constant

- The sum of K.E. and P.E. is the total energy
- ∴ This shows that the total energy of the particle is conserved.

1.3 System of Particles:

(a) Center of Mass:

Center of mass is a point inside or outside the body at which total mass of a ~~system~~ body is assumed to be concentrated.



Let m_1 and m_2 be two point masses at distance r_1 and r_2 respectively from the origin O.

Now R be the distance from origin O and C.M. where whole mass of the sys. i.e. $m = m_1 + m_2$ is supposed to be concentrated.

Equating mass moments about O we get.

$$(m_1 + m_2) R = m_1 r_1 + m_2 r_2$$

$$R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$R = \frac{m_1 r_1 + m_2 r_2}{m} \quad \because m = m_1 + m_2$$

Similarly, if there are n particles in the system then C.M. of these particles relative to origin is

$$R = \frac{m_1 r_1 + m_2 r_2 + \cdots + m_n r_n}{m_1 + m_2 + \cdots + m_n}$$

$$R = \frac{\sum_i m_i r_i}{\sum_i m_i} = \frac{\sum_i m_i r_i}{M}$$

Where, $M = \sum_i m_i$ = total mass of the system.

$R = (x_{c.m.}, y_{c.m.}, z_{c.m.})$ and $r_i = (x_i, y_i, z_i)$

Physical Significance of C.M. :-

We know the position of the center of mass from the origin.

$$\vec{R} = \frac{\sum_i m_i \vec{r_i}}{M}$$

i.e. $M\vec{R} = \sum_i m_i \vec{r_i}$

$$M\vec{R} = m_1\vec{r_1} + m_2\vec{r_2} + \cdots + m_n\vec{r_n}$$

Differentiating w.r.t. t

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r_1}}{dt} + m_2\frac{d\vec{r_2}}{dt} + \cdots + m_n\frac{d\vec{r_n}}{dt}.$$

$$\therefore \quad M V_{c.m.} = m_1\vec{v_1} + m_2\vec{v_2} + \cdots + m_n\vec{v_n}$$

where, V_{cm} = the velocity of C.M.

V_i = the velocities of the particles in the system.

By diff. the above expression w.r.t. 't'

we get, $M\frac{d\vec{V_1}}{dt} = m_1\frac{d\vec{V_1}}{dt} + m_2\frac{d\vec{V_2}}{dt} + \cdots + m_n\frac{d\vec{V_n}}{dt}$

$$M\vec{a}_{cm} = m_1\vec{a_1} + m_2\vec{a_2} + \cdots + m_n\vec{a_n}$$

$$F_{ext} = \vec{f_1} + \vec{f_2} + \cdots + \vec{f_n}$$

———(from Newtons Law)

* ① There are two types of forces Internal and external.

② The internal forces do not contribute to the motion as they are equal and opposite.

Hence internal forces are neglected

③ Therefore, total force acting on the system is only the total external force. i.e. $\vec{F} = F_{ext}$

Thus, the sys of particles or body moves when all the external forces are applied at the C.M.

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(1) A force of 100N produces an acceleration of 2.5 m/s² of a body. What must be the mass of the body?

→ $F = 100N$ $a = 2.5\ m/s^2$

We know, $F = ma$

$$m = \frac{F}{a} = \frac{100}{2.5} = 40kg.$$

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(2) A body of mass 50 kg moving with speed of 30m/s collides with another body of mass 200kg which is initially at rest. After collision both the bodies stick to each other and move. Find the common velocity.

→ $m_1 = 50kg$ $u_1 = 30\ m/s$

$m_2 = 200kg$ $u_2 = 0\ m/s$

According to the law of conservation of linear momentum,

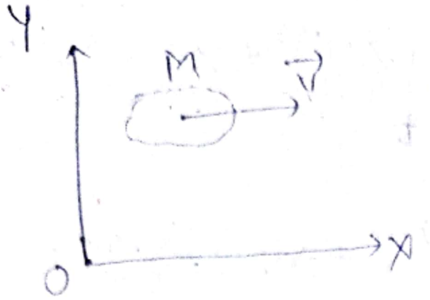
$$m_1u_1 + m_2u_2 = (m_1 + m_2)V$$

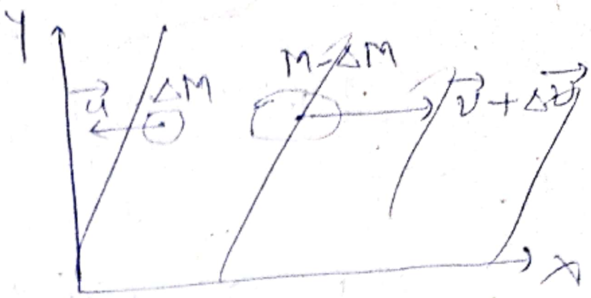
$$50 \times 30 + 200 \times 0 = (50 + 200)V$$

$$1500 = 250V$$

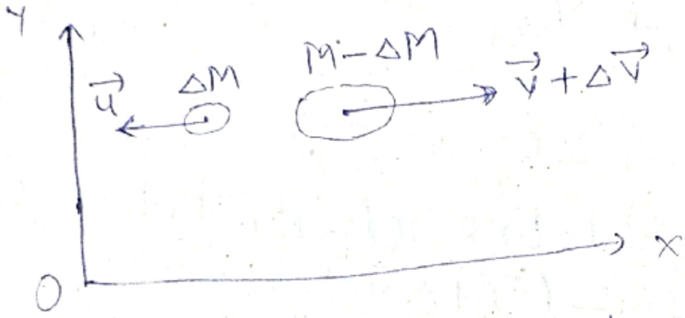
$$V = \frac{1500}{250} = 6\ m/s.$$

1.7 Motion of Rocket:





Let us consider a rocket system of mass M moving with a velocity $\vec{v}$ at time "t"



The fuel in the roket is burnt and gas is ejected continuously.

Let at time $(t+\Delta t)$, ΔM be the mass of fuel ~~gas~~ gas whose C.M. moving with a velocity $\vec{u}$

Now the mass of rocket reduced to $(M-\Delta M)$ and its velocity increased to $(\vec{v}+\Delta\vec{v})$

If $\vec{F}$ is the external force acting on the system, then by Newton's sec. Law of motion, we have,

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{P_f - P_i}{\Delta t}$$

where, P_i = Initial momentum of the sys. $= M\vec{v}$

P_f = final momentum of the system at time $(t+\Delta t)$

$= (M-\Delta M)(V+\Delta V) + \Delta M\vec{u}$

$$\therefore \vec{F} = \frac{[(M-\Delta M)(\vec{v}+\Delta\vec{v}) + \Delta M\vec{u}] - M\vec{v}}{\Delta t}$$

$$= \frac{[\cancel{M\vec{v}} + M\Delta\vec{v} - \Delta M(\vec{v}+\Delta\vec{v}) + \Delta M\vec{u} - \cancel{M\vec{v}}}{\Delta t}$$

$$= M\frac{\Delta\vec{v}}{\Delta t} + \vec{u}\frac{\Delta M}{\Delta t} - (\vec{v}+\Delta\vec{v})\cdot\frac{\Delta M}{\Delta t}$$

$$= M\frac{\Delta v}{\Delta t} + [\vec{u} - (\vec{v}+\Delta\vec{v})]\frac{\Delta M}{\Delta t}$$

$$\vec{F} = M\frac{\Delta v}{\Delta t} + V_{rel}\frac{\Delta M}{\Delta t}.$$

where, ~~$\vec{v}_{rel} = \vec{v} - (\vec{v}+\Delta\vec{v})$~~

$\vec{v}_{rel}$ = relative velocity of the ejected gas.

$= \vec{v} - (\vec{v}+\Delta\vec{v})$

$\therefore$ As $\Delta t \to 0$ then we get,

$$\vec{F} = M\frac{dv}{dt} + V_{rel}\frac{dM}{dt}$$

where, $V_{rel}\frac{dM}{dt}$ = the thrust experienced on the rocket

$\frac{dM}{dt}$ is negative, as the mass of the roket decreases with time.

$$\therefore \vec{F} = M\frac{dv}{dt} - V_{rel}\frac{dM}{dt}$$

$$\therefore M\cdot\frac{dv}{dt} = \vec{F} + V_{rel}\frac{dM}{dt}$$

where, $\frac{d\vec{v}}{dt}$ is the acceleration of the rocket

$$\therefore M\frac{dv}{dt} = Mg + V_{rel}\frac{dM}{dt}$$

Dividing by M

$$\frac{dv}{dt} = g + \frac{V_{rel}}{M}\frac{dM}{dt}$$

multiply by 'dt'

$$dv = g\,dt + V_{rel}\frac{1}{M}dM$$

Integrating Both sides

$$v = gt + V_{rel}\ln\frac{M_t}{M_0}$$

This eqn gives vel. of rocket after time t in the presence of gravitational pull.

2) Discuss the working Principle of a rocket.

$\boxed{\text{or}}$

Discuss the rocket motion.