2. Ordinary Differential Eqn [2023-24]

2.1 : Introduction

* Variable quantities : The quantities which changes with changing another quantity.

Independent Variables :
The variables independent of other variables or quantities.

Dependent Variables :
The variables which depend on other variables or quantities.

Derivative :
Variation of a physical quantity (variable) w.r.t. another is called derivative.

2.2. Differential Equation :

Eqn involving – independent and dependent variables
– And derivative of dependent variables w.r.t. independent variable

e.g. If we have a function,

$$y = f(x)$$

then, x is independent variable
y is dependent variable

∴ the derivative of y w.r.t. x are

$y' = \frac{dy}{dx}$ ---- (first order derivative)

$y'' = \frac{d^2y}{dx^2}$ ---- (Sec. order derivative)

$$y''' = \frac{d^3y}{dx^3}$$ ---- (third order derivative)

* Differential eq^n play an important role in science and technology.
(e.g. Differential eq^ns have indispensable role)
In the study of oscillations of mechanical and electrical problems, conduction of heat. etc.

(a) Order of differential Eq^n :

It is the highest order of the derivative involved in the differential equation.

① $\frac{dy}{dx} = \sin x$ ---------- first order diff. eq^n.

② $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ ---------- first order diff. eq^n

③ $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ ---------- Sec. order diff. eq^n

④ $\frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^3}$ ---------- Sec. order diff. eq^n

Other e.g.

① $\frac{d^3y}{dx^3} - 3x\left(\frac{dy}{dx}\right)^3 + 2y = x^2$ ---------- (third order diff. Eq^n)

② $\left(1+\frac{d^2y}{dx^2}\right)^{3/2} = a\frac{d^2y}{dx^2}$ ---------- (Sec. ~~order~~ order diff Eq^n)

③ $P\frac{dy}{dx} + Qy = R$ ---------- (first order diff. eq^n)

④ $5\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5\int y\,dx = 5x$

Integrating the above Eq^n

$5\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5xy = 5x$ ---------- (Sec. order diff Eq^n)

(b) Degree of differential Eqn :

- The power of highest order derivative present in the eqn
- when fractional powers of all derivatives have been removed.

* Conditions for writing Degree of diff. eqn

- degree is +ve degree Indices
- Not in fraction / radical sign
- Not in the denominator

e.g. (1) $\frac{dy}{dx} = \sin x$ ------ (degree 1)

(2) $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ ------ (degree 1)

(3) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ ------ (degree 1)

(4) $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$ ------ (degree 2)

Other eg.

(1) $y = \sqrt{x}\,\frac{dy}{dx} + \frac{K}{dy/dx}$

$y\frac{dy}{dx} = \sqrt{x}\left(\frac{dy}{dx}\right)^2 + K$ ------ (degree 2)

(2) $y = x\frac{dy}{dx} + K\left[\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right]$

$\left(y - x\frac{dy}{dx}\right)^2 = K^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$ ------ (degree 2)

(3) $\frac{d^2y}{dx^2} - 3x\left(\frac{dy}{dx}\right)^6 + 2y = x^2$ ------ (degree 1)

c) Linear differential Equation:

→ The dependent variable and its derivatives occur only in the first degree and are not multiplied together.

The standard forms of first and second order linear ordinary diff. eqn are

I] First order

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Where, $P(x)$ & $Q(x)$ are only function of x

e.g. $\frac{dy}{dx} = \sin x + \cos x$ ------ Linear diff. Eqn

$\cancel{\frac{d^2x}{dt^2} = -kx}$ ------ Linear diff. Eqn

$\left(\frac{dy}{dx}\right)^2 + xy = x^2$ ------ Non-linear diff. Eqn
(* Because, degree is not first degree)

II] Sec. Order:

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = f(x)$$

Where, $p(x)$, $q(x)$ & $f(x)$ are only function of x

e.g. $\frac{d^2x}{dt^2} = -kx$ ------ linear diff. Eqn

(d) Homogeneous Diff. Eqn

We know, $\frac{dy}{dx} + P(x) = Q(x)$ — (first order diff. Eqn) — ①

$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + q(x)\,y = f(x)$ — (sec. order diff. Eqn) — ②

If right hand side of eqn ① and ② (i.e. $Q(x) = 0$ and $f(x) = 0$), then the corresponding diff. eqn are said to be Homogeneous.

i.e. $\frac{dy}{dx} + P(x)\,y = 0$

and $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + q(x)\,y = 0$

* Non-Homogeneous Differential Eqn :

If right hand sides of eqn ① and ② (i.e. $Q(x) \neq 0$ and $f(x) \neq 0$), then the corresponding differential eqn are said to be non-homogeneous.

2.3 Types of Differential Equations:

(a) Ordinary differential equations

(b) Partial differential equations.

(a) Ordinary Differential Equations:

— It involves total derivatives (i.e. there is only one independent variable)

e.g. 1. $\frac{dy}{dx} = \sin x$

2. $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

3. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

4. $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$

(b) Partial Differential Equation:

— It contains two or more independent variables and partial derivatives w.r.t. those two or more independent variables.

1. $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 4z$ ——— where $z = z(x, y)$

2. $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \phi}{\partial t^2}$ ——— where $\phi = \phi(x, t)$

2.4. First Order Homogeneous Differential Equations:

The general form of first order homogeneous differential eqn is ~~given by~~

$$\frac{dy}{dx} + p(x)\, y = 0$$

* The soln of this Eqn is obtained by variable separable form.

$$\therefore \quad \frac{dy}{dx} = -p(x)\, y$$

$$\therefore \quad \frac{dy}{y} = [-p(x)]\, dx$$

$$\text{i.e.} \quad \frac{1}{y}\, dy = [-p(x)]\, dx$$

$$f(y)\, dy = g(x)\, dx$$

The general soln is obtained By integrating above Eqn.

$$\int f(x)\, dx = \int g(y)\, dy + C,$$

where C is an arbitary constant.

– The general soln is obtained by integrating above eqn once.

– The value of arbitary constant can be determined from the initial conditions of the problem.

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~~e) Obtain an expression for the instantaneous current after the ckt. is switched off.~~

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2) Solve the differential Eqn: $(1+x^2)dy - xy\,dx = 0$

$\rightarrow$ $(1+x^2)dy - xy\,dx = 0$

$\frac{dy}{y} = \frac{x}{1+x^2}dx$ ——— variables separated

$\therefore$ ~~$\frac{dy}{y}$~~ Now, on integration, we get,

$\int \frac{dy}{y} = \int \frac{x}{1+x^2}dx + C_1$ where, C_1 is integration constant

$\ln y = \frac{1}{2}\int \frac{2x}{1+x^2}dx + C_1$

$\ln y = \frac{1}{2}\ln(1+x^2) + \ln C$

$\ln y = \ln\sqrt{1+x^2} + \ln C$

$\ln y = \ln C\sqrt{1+x^2}$

$\therefore$ $y = C\sqrt{1+x^2}$ is the required solution.

~~4) Solve: $x\frac{dy}{dx} + y = x^2 + 1$~~

$\rightarrow$

8) solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\longrightarrow$ $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\frac{dy}{dx} = (e^x + x^2) e^{-y}$

$\therefore \frac{dy}{e^{-y}} = (e^x + x^2) dx$

$e^y dy = (e^x + x^2) dx$

On integrating we get,

$e^y = e^x + \frac{x^3}{3} + C$

$e^y = \left(e^x + \frac{x^3}{3}\right) + C$ is the required Solution

1) N_0 is the number of radioactive nuclei at $t=0$, then find an expression for the number N left at any time (t)

$\longrightarrow$ We know $\frac{dN}{dt} = -\lambda N$ ------ Radioactive decay eqn

$\lambda \rightarrow$ decay constant

-ve sign $\rightarrow$ N decreases with increasing (t).

$\frac{dN}{dt} + \lambda N = 0$

$I.F. = e^{\int P(x) dx} = e^{\int \lambda dt} = e^{\lambda t}$

$\therefore N \cdot I.F. = \int Q \cdot I.F. + C$

$N \cdot e^{\lambda t} = 0 + C$ $\quad \{\because Q(t) = 0$

$N = C \cdot e^{-\lambda t}$

At $t = 0$ $N = N_0$

$\therefore N_0 = C e^{-\lambda(0)}$

$N_0 = C e^{0}$

$N_0 = C$

$\therefore N = N_0 e^{-\lambda t}$

Extremly Quickly

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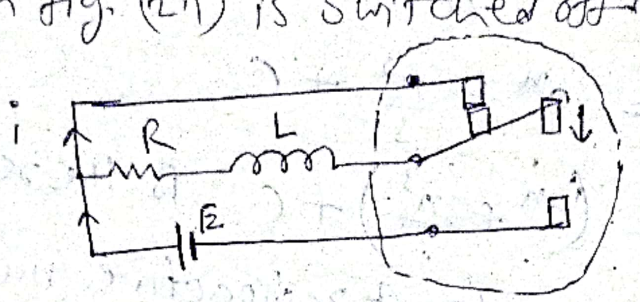
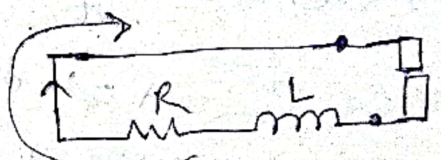
e.g. Obtain an expression for the instaneous current after the ckt. in fig. (2.1) is switched off.



Moarse key

$\longrightarrow$ The ckt. is switched off at time $t = 0$ then $i = i_0$

$\therefore$ By Kirchoff's Volt Law



$$V_1 + V_2 = 0$$

$$\cancel{iR + L\frac{di}{dt} = 0}$$

$$L\frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0 \quad \text{—— (standard homogeneous Eq}^n\text{)}$$

$\therefore$ on separating the variables, we get:

$$\frac{di}{i} = -\frac{R}{L}\,dt$$

∴ On integrating, we get

$$\ln i = -\frac{R}{L}t + \ln c$$ ---- where $\ln c$ is a const.

$$\ln i - \ln c = -\frac{R}{L}t$$

$$\ln i/c = -\frac{R}{L}t$$

$$\frac{i}{c} = e^{-R/L\,t}$$

$$i = c\,e^{-\frac{R}{L}t}$$

To ~~oten~~ obtain the constant c, we use initial conditions

At $t=0$ $\quad i = i_0$

$$i = c e^{-\frac{R}{L}t}$$

$$i_0 = c\,e^0$$

$$i_0 = c$$

$$\cancel{\therefore\ i = i_0\, e^{-R/L\,t}}$$

$$i = i_0\ e^{-\frac{R}{L}t}$$

i.e. the current in the ckt. decreases exponentially from i_0 to become zero at $t=\infty$

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2.5. ~~Sec. Order Homogeneous Diff. Eqn With Constant Coefficients:~~

~~General form:~~

$$\cancel{\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} +}$$

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* First order differential Eqn:

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

General soln is $\quad y \cdot I.F. = \int Q(x)\, I.F.\, dx + C$

$$\because I.F. = e^{\int P(x)\,dx}$$

Page-52 Problem (4) Solve, $x\frac{dy}{dx} - \frac{1}{2}y = x+1$

$\rightarrow$

$$x\frac{dy}{dx} - \frac{1}{2}y = x+1$$

$$\frac{dy}{dx} - \frac{1}{2x}y = \left(1+\frac{1}{x}\right)$$

Here $P(x) = -\frac{1}{2x}$

$$\therefore I.F. = e^{\int P(x)\,dx} = e^{\int -\frac{1}{2x}} = e^{-\frac{1}{2}\ln(x)} = e^{\ln x^{-1/2}} = x^{-1/2} = \frac{1}{\sqrt{x}}$$

$\therefore$ The General soln of above eqn is

$$y \cdot I.F. = \int Q(x)\, I.F.\, dx + C$$

$$y \cdot \frac{1}{\sqrt{x}} = \int \left(1+\frac{1}{x}\right)\cdot\frac{1}{\sqrt{x}}\,dx + C$$

$$y \cdot \frac{1}{\sqrt{x}} = \int \left(x^{-1/2} + x^{-3/2}\right) dx + C$$

$$y \cdot \frac{1}{\sqrt{x}} = \left(\frac{x^{1/2}}{1/2} + \frac{x^{-1/2}}{-1/2}\right) + C$$

$$y \cdot \frac{1}{\sqrt{x}} = \left(2\sqrt{x} - \frac{2}{\sqrt{x}}\right) + C\sqrt{x}$$

$$y = 2x - 2 + C\cdot\sqrt{x}$$

2.5. Sec. Order Homogeneous Diff. Eqⁿ With Constant Coefficient :-

General form:

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + q(x)\,y = 0$$

This eqⁿ with constant coefficients is

$$\frac{d^2y}{dx^2} + A\frac{dy}{dx} + By = 0$$

Where, A and B are constants.

if $D = \frac{d}{dx}$, is linear operator

then $\frac{d}{dx} = D$ & $\frac{d^2}{dx^2} = D$

$$\cancel{\frac{d^2}{dx^2} = D^2}$$

$$\therefore (D^2 + AD + B)\,y = 0$$

We get, $D^2 + AD + B = 0$

$\therefore$ The roots are, $\left\{ \because \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right.$

$$a = \frac{-A + \sqrt{A^2 - 4B}}{2} \qquad b = \frac{-A - \sqrt{A^2 - 4B}}{2}$$

Case I: The roots are real and unequal $(a \neq b)$

If a and b are real and unequal roots,

then, $(D^2 + AD + B)y = 0$

$(D-a)(D-b)y = 0$

$(D-a)y_1 = 0$ and $(D-b)y_2 = 0$

$y_1 = c_1 e^{ax}$ $y_2 = c_2 e^{bx}$

$\therefore$ General solⁿ of Eqⁿ of eqⁿ $\frac{d^2y}{dx^2} + A\frac{dy}{dx} + By = 0$ is

$$y = y_1 + y_2$$

$$\therefore \quad y = c_1 e^{ax} + c_2 e^{bx}$$

Where c_1 and c_2 are arbitary constants.

Case II: The roots are real and equal $(a = b)$

If a and b are real and ~~un~~equal roots

then, $(D^2 + AD + B)y = 0 \Rightarrow (D-a)(D-b)y = 0$

$\Rightarrow (D-a)(D-a)y = 0$ $\{\because a = b\}$ —— (1)

One of the solⁿ of above eqⁿ $(D-a)y = 0$ is

$$y_1 = c_3 e^{ax}$$

to obtain linearly independent sec. solⁿ

Let us put $(D-a)y = x$ —— (2)

$\therefore$ eqⁿ (1) becomes

$(D-a)x = 0$ —— (3)

$\therefore$ Sol^n of eq^n 3 is

$$x = c_4 e^{ax}$$

$\therefore$ eq^n 2 becomes,

$$(D-a)\,y = x$$

$$(D-a)\,y = c_4 e^{ax}$$

$$\frac{dy}{dx} - ay = c_4 e^{ax}$$

This is first order diff. eq^n whose sol^n is obtained as follows,

$$P.I. = I = e^{-\int p(x)dx} = e^{-\int a\,dx} = e^{-ax}$$

$\therefore$ The general sol^n is

$$y.\ P.I = \int Q(x)\,.\ P.I\ dx + c_5$$

$$y\ e^{-ax} = \int c_4\, \cancel{e^{ax}}\,.\,\cancel{e^{-ax}}\ dx + c_5$$

$$y.\ e^{-ax} = c_4 \int dx + c_5$$

$$y.\ e^{-ax} = c_4 x + c_5$$

$$y = (c_4 x + c_5)\, e^{ax}$$

This is the general solution of the sec. order diff. eq^n when roots are real and equal.

Case III : If roots are complex conjucate pair.

If $a = \alpha + i\beta$ and $b = \alpha - i\beta$ are the roots of auxiliary eqn, then the general soln of eqn $(D^2 + AD + B)y = 0$ is

$$y = c_6 e^{(\alpha + i\beta)x} + c_7 e^{(\alpha - i\beta)x}$$

$$= e^{\alpha x}\left[c_6 e^{i\beta x} + c_7 e^{-i\beta x}\right]$$

$$= e^{\alpha x}\left[c_6(\cos\beta x + i\sin\beta x) + c_7(\cos\beta x - i\sin\beta x)\right]$$

$$= e^{\alpha x}\left[(c_6 + c_7)\cos\beta x + i(c_6 - c_7)\sin\beta x\right]$$

$$y = e^{\alpha x}\left[A_1 \cos\beta x + B_1 \sin\beta x\right]$$

where, $A_1 = (c_6 + c_7)$ and
$B_1 = (c_6 - c_7)$

* Problems (16) * auxiliary Eqn. → reduced Eqn.

I] Roots are Real and Distinct

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$

→ ~~The auxiliary eqn.~~ Using Diff. operator $D = \frac{d}{dx}$ and $D^2 = \frac{d^2}{dx^2}$

$(D^2 - D - 6)y = 0$

The auxiliary eqn is

$(D^2 - D - 6) = 0$ or $y = 0$ — ~~(the auxiliary eqn.)~~

$\therefore (D-3)(D+2) = 0$

$\therefore$ Roots are $D = -2, 3$

$\therefore$ Soln of diff eqn

$y = c_1 e^{-2x} + c_2 e^{3x}$

② The current 'i' flowing through a ckt. is given by eqn

$\frac{d^2i}{dt^2} + 60\frac{di}{dt} + 500i = 0$, solve for the current i at time > 0

→ ~~The auxiliary eqn~~ Use Diff. operator $D = \frac{d}{dx}$ and $D^2 = \frac{d^2}{dx^2}$

$(D^2 + 60D + 500)i = 0$

The auxiliary eqn is

$(D^2 + 60D + 500) = 0$ or $i = 0$

$\therefore (D+10)(D+50) = 0$

$D = -10, -50$

$\therefore$ Soln of diff eqn

$i = c_1 e^{-10t} + c_2 e^{-50t}$

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II] Equal and Real Roots:

① Solve the differential eqⁿ $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

→ ~~The auxiliary eqⁿ is~~ Use Diff. operator $D = \frac{d}{dx}$ and $D^2 = \frac{d^2}{dx^2}$

$$(D^2 - 4D + 4)\,y = 0$$

The auxiliary eqⁿ is

$$(D^2 - 4D + 4) = 0 \quad \text{or } y = 0$$

$$\therefore (D-2)(D-2) = 0$$

∴ Roots are $D = 2, 2$

∴ Solⁿ of Diff. eqⁿ is

$$y = (c_1 + c_2 x)\, e^{2x}$$

② $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$

→ ~~The auxiliary eqⁿ is~~ $\cancel{(D^2 - 8D + 16)\,y = 0}$ $\cancel{(D^2 - 8D + 16) = 0 \text{ or } y = 0}$ Use Diff. operator $D = \frac{d}{dx}$ & $D^2 = \frac{d^2}{dx^2}$

$$(D^2 - 8D + 16)\,y = 0$$

The auxiliary eqⁿ is

$$D^2 - 8D + 16 = 0$$

$$\therefore (D-4)(D-4) = 0$$

∴ Roots are $D = 4, 4$

∴ Solⁿ of diff. eqⁿ is

$$y = (c_1 + c_2 x)\, e^{4x}$$

III] Roots are complex

① Find the general sol^n of $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

→ The auxilary eq^n is.

$$D^2 + D + 1 = 0$$

$\therefore$ $m_1 = \frac{-1 + \sqrt{1-4}}{2}$ $\quad$ $m_2 = \frac{-1 - \sqrt{1-4}}{2}$

$m_1 = -\frac{1}{2} + \frac{\sqrt{-3}}{2}$ $\quad$ $m_2 = -\frac{1}{2} - \frac{\sqrt{-3}}{2}$

$m_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$ $\quad$ $m_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

The general sol^n is

$$y = e^{-x/2}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right)$$

② Solve: $\frac{d^2y}{dx^2} + 9y = 0$

→ The auxiliary eq^n is

$$D^2 + 9 = 0$$

The roots are

$D = -9$

$D = 9i^2$

$D = \pm 3i$

$D = -3i, 3i$

or $D = \pm 3i = (\alpha \pm 3i)$ $\quad$ $\therefore \alpha = 0$ $\beta = 3$

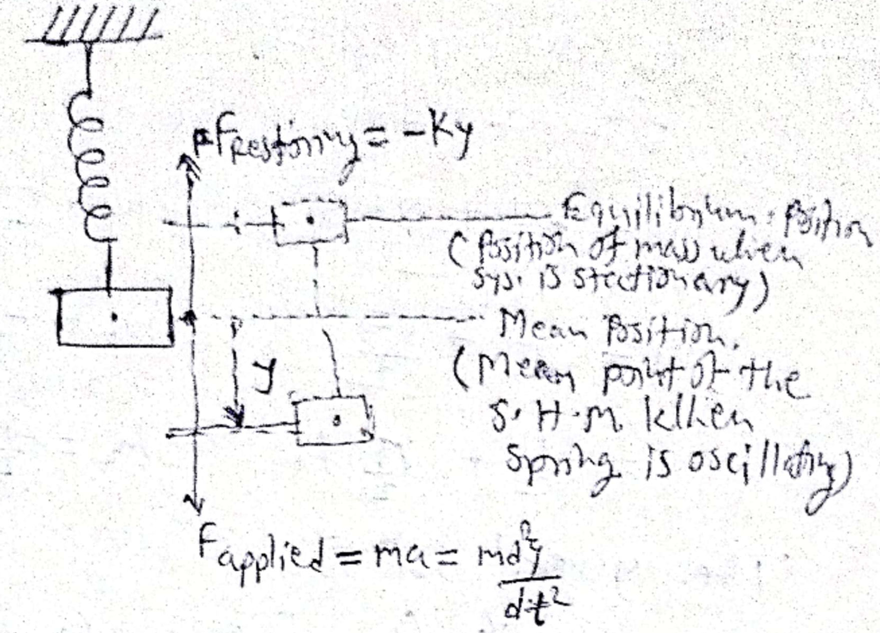
$\therefore$ The general sol^n is

$$y = e^{\alpha x}[A \cos \beta x + B \sin \beta x]$$

$$y = A \cos 3x + B \sin 3x \quad (\because \alpha = 0 \text{ and } \beta = 3)$$

Where A and B are arbitary constant.

Example: Consider a mass(m) attached to a spring with spring constant(K)



The mass in equilibrium position when displaced through a dist. y and realised, performs simple harmonic oscillations, represented by the differential eqn

i.e. Fapplied = Frestoring

$$m\frac{d^2y}{dx^2} = -Ky$$

$$\therefore m\frac{d^2y}{dx^2} + Ky = 0$$

$$\frac{d^2y}{dx^2} + \frac{K}{m}y = 0$$

$$\frac{d^2y}{dx^2} + w^2y = 0 \qquad \left\{\because w^2 = \frac{K}{m}\right.$$

$\therefore$ the auxiliary Eqn is

$$(D^2 + w^2) = 0$$

$\therefore$ Roots are $D = \pm iw$

$\therefore$ General soln is

$$y = Ae^{iwx} + Be^{-iwx}$$

$$y = A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t)$$
$$= (A+B)\cos\omega t + i(A-B)\sin\omega t$$
$$y = A_1 \cos\omega t + B_1 \sin\omega t.$$

Where A_1 & B_1 are arbitary constants

* The period (T)

The period (T) of the oscillation is determined by the condition.

$$y(t) = y(t+T)$$

$$\therefore A_1 \cos\omega t + B_1 \sin\omega t = A_1 \cos\omega(t+T) + B_2 \sin\omega(t+T)$$
$$= A_1[\cos\omega t \cdot \cos\omega T - \sin\omega t \cdot \sin\omega T]$$
$$+ B_1[\sin\omega t \cdot \cos\omega T + \cos\omega t \cdot \sin\omega T]$$

The above eqn is valid only if

$\cos\omega T = 1$ and $\sin\omega T = 0$

$$\therefore \omega T = 2\pi n$$

$$T = \frac{2\pi n}{\omega} = 2\pi n\sqrt{\frac{m}{K}} \qquad \because \omega = \sqrt{\frac{K}{m}}$$

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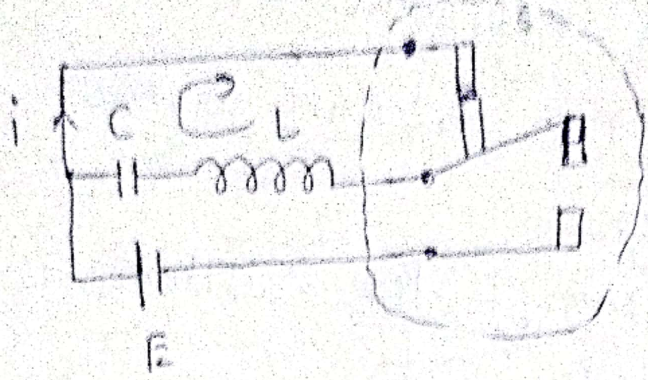
Problem 7: Discuss the nature of charge in a ckt. containing a capacitor (C) and an inductor (L) after removing the charging source i.e. battery (E)

→



charging–discharging key

$$V_1 + V_2 = 0$$

$$L\frac{di}{dt} - \frac{q}{C} = 0$$

$$L\frac{d^2q}{dt^2} - \frac{q}{C} = 0$$

$$\frac{d^2q}{dt^2} - \frac{q}{LC} = 0$$

$$\frac{d^2q}{dt^2} - \omega^2 q = 0 \qquad \because \omega = \frac{1}{\sqrt{LC}}$$

~~Usin~~ Using operator $D = \frac{d}{dt}$ $D^2 = \frac{d^2}{dt^2}$ we get

The auxiliary eqn.

$$D^2 - \omega^2 = 0$$

$$D = \pm \omega$$

$$q = A e^{\omega t} + B e^{-\omega t} \qquad (\because A \,\&\, B \text{ are constants})$$

* The charge oscillates in ckt. with period T given by.

$$\omega = \frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$$

$$\therefore T = 2\pi\sqrt{LC}$$

* Freq. of oscillating charge $= n = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}$