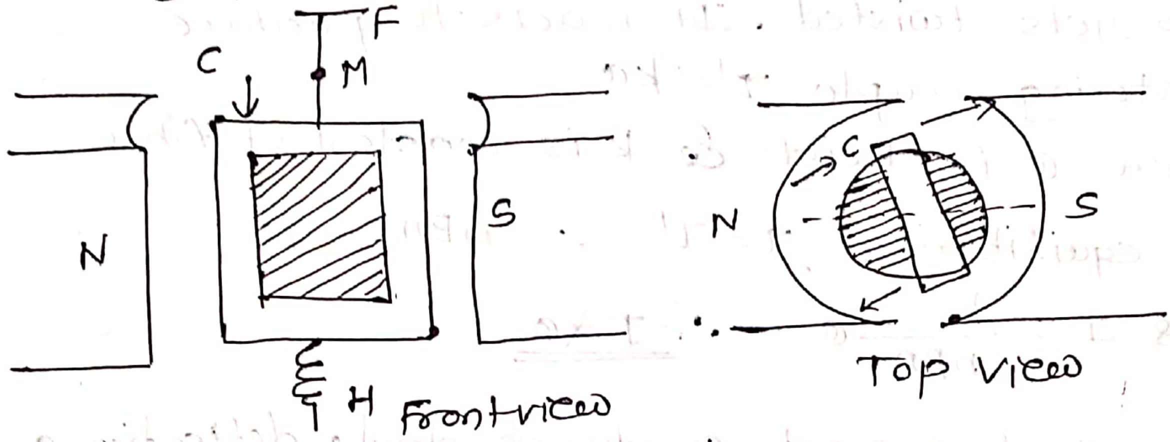
3. Ballistic Galvanometer

A galvanometer is a device to measure small electric current.

* Construction & Working of Ballistic Galvanometer:–



Moving coil Galvanometer

① Ballistic Galvanometer consists of coil (C). Coil C is insulated thin wire wound round on light rectangular frame in large number of turns.

② The coil is suspended in radial magnetic field of strong magnet (NS) having cylindrically concave pole pieces.

③ The coil suspended by means of conducting phosphorbronz suspension (F) & supported from below by ~~means of~~ helix (H).

④ A small mirror (M) is attached to fibre.

⑤ An iron core (I) is arranged at centre of coil.

⑥ The mirror helps to measure the deflection of of galvanometer by lamp & scale arrangement.

(7) The iron core helps to increase magnetic induction & thereby sensitivity of galvanometer.

If a steady current I is passed through coil, it produces deflecting couple $\tau = nBAI$.

Due to couple, coil turns & suspension fibre gets twisted. It reacts to produce restoring couple $\tau' = k\theta$

where θ is twist & k is constant of fibre

In equilibrium, $\tau = \tau'$ $\therefore$ $nBAI = k\theta$

Thus $I = \frac{k}{nBA}\theta$ $\therefore$ $\underline{\underline{I \propto \theta}}$

Here, steady current produces steady deflection θ.

* Expression for charge flowing through Ballistic Galvanometer:-

The ballistic galvanometer are used to measure charge.

<u>Theory</u>:- (1) If a current I is passed through the coil of n turns, length l & breadth b, & if B is magnetic induction of field around it, the force acting on length side conductor of coil is

$F = nBI'l$

If this current flows for a short interval of time dt, the impulse produced

$= \text{force} \times \text{time} = F\,dt = nBI'l\,dt$

(Impulse - change in momentum)

But impulse brings about change in momentum,

$\therefore$ dp = change in momentum = $nBI'l\,dt$

$= nBl \cdot I'dt$

But $I' = \dfrac{dq}{dt}$ $\therefore$ ~~dp = n~~ $\Rightarrow I'dt = dq$

$\therefore$ $dp = nBl\,dq$

Therefore, change in momentum for a charge q is,

$p = \int dp = \int_0^q nBl \cdot dq$

$\therefore p = nBlq \longrightarrow (1)$

Now, ~~angular~~ Linear momentum produced in coil,

$L = rp = p \times b \quad \ldots (r = b)$

$\therefore$ $L = nBlq \cdot b$

But $l \cdot b$ = area of coil = A

$\therefore$ $L = nBqA \longrightarrow (2)$

But $L = I_0\omega$ = M. I. of coil $\times$ angular speed

$\therefore$ $I_0\omega = nBAq \longrightarrow (3)$

(II) The restoring couple due to twist in fibre is, $\tau' = k\theta$ where k = couple per unit twist

Now, work done in twist $d\theta$ = $dW = \tau' d\theta$

$\therefore$ $dW = k\theta\,d\theta$

The work done in twisting the fibre is

$\int dW = \int_0^\theta k\theta\,d\theta$

$$\therefore W = \frac{1}{2} k\theta^2 \longrightarrow (4)$$

Work done is equal to kinetic energy $E_K = \frac{1}{2} I_o \omega^2$ for deflection θ.

$$\therefore \frac{1}{2} I_o \omega^2 = \frac{1}{2} k\theta^2$$

$$\therefore I_o \omega^2 = k\theta^2 \longrightarrow (5)$$

(iii) The time period of oscillation of coil, due to twist (torsion) in fibre, is

$$T = 2\pi \sqrt{\frac{I_o}{k}}$$

$$\therefore I_o = \frac{T^2 k}{4\pi^2} \longrightarrow (6)$$

~~Multiply~~ Multiply eqn (5) & (6), we get

$$\therefore I_o \cdot I_o \omega^2 = \frac{T^2 k}{4\pi^2} k \cdot \theta^2$$

$$\therefore I_o^2 \omega^2 = \frac{T^2 k^2}{4\pi^2} \theta^2$$

$$\therefore I_o \omega = \frac{T k \theta}{2\pi} \longrightarrow (7)$$

Compare eqn (3) & (7), we get

$$nBAq = \frac{Tk\theta}{2\pi}$$

$$\therefore q = \frac{T}{2\pi} \cdot \frac{k\theta}{nAB} \longrightarrow (8)$$

The quantity $\frac{Tk}{2\pi nAB}$ is constant & is called ballistic reduction factor.

$\therefore q = \text{constant} \times \theta \longrightarrow (9)$

$\therefore \underline{q \propto \theta}$

Now for a moving coil galvanometer,

$$I = \frac{k\theta}{nBA} \Rightarrow dI = \frac{k}{nBA} d\theta$$

$$\therefore \frac{dI}{d\theta} = \frac{k}{nBA} \longrightarrow (10)$$

But value of $\frac{k}{nBA}$ in eq^n (8),

$$\therefore \boxed{q = \frac{T}{2\pi} \cdot \frac{dI}{d\theta} \cdot \theta} \longrightarrow (11)$$

Hence the measurement of charge q done in three steps:

1. The galvanometer is given free oscillations & time period T of oscillation is measured.
2. For various known steady currents I, the steady deflections θ are measured. The graph of I (Y-axis) against θ (X-axis) gives straight line. Its slope gives $\frac{dI}{d\theta}$.
3. charge q to be measured when deflection θ of galvanometer is noted.

By using formula, $q = \frac{T}{2\pi} \frac{dI}{d\theta} \cdot \theta$

$$\therefore \boxed{q = \frac{T}{2\pi} \times \text{slope} \times \theta} \longrightarrow (12)$$

$\therefore$ The charge q through galvanometer is calculated.

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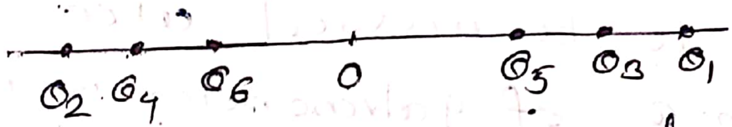
* Correction for Damping in Galvanometer:-

The decrease in the deflection of the galvanometer due to the resistance of the air the electromagnetic field in which the coil oscillates is called damping.

The damping is due to – ① opposition of the air surrounding the coil, it is called air damping. ② resistance or opposition due to the induced current in coil when it oscillates in the magnetic field, the induced current introduces a torque (couple) opposing deflecting torque due to charge q. This torque is in addition to restoring torque produced due to twist in suspension fibre. It is called electromagnetic damping.

Let θ be the true throw in absence of damping & $\theta_1, \theta_2, \theta_3, \ldots$ be successively observed throws to the right & left.



Throws due to damped oscillation

Here $\dfrac{\theta_1}{\theta_2} = \dfrac{\theta_3}{\theta_4} = \dfrac{\theta_3}{\theta_4} = \ldots \; d$ say $\longrightarrow ①$

d is called decrement.

Let $d = e^{\lambda}$, therefore $\lambda = \log_e d$

λ is called logarithmic decrement.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$$

Each complete oscillation is formed by the two successive throws on same side, θ_1 to θ_3, θ_3 to θ_5, ... & hence each complete oscillation contains of two successive swings, from θ_1 to θ_2, θ_2 to θ_3, etc. Hence θ_1 to θ_2, θ_2 to θ_3 forms half oscillation.

Hence, $\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d \times d = d^2 = (e^\lambda)^2 = e^{2\lambda}$

.... [from ①] $\longrightarrow$ ②

Let θ be the first true throw in absence of damping. When charge q is passed, the coil starts deflecting & tries to reach the amplitude θ, but when comes to end of throw, i.e. it completes quarter of oscillation, its amplitude becomes θ_1 due to damping.

$$\therefore \frac{\theta}{\theta_1} = d^{1/2} = (e^\lambda)^{1/2} = e^{\lambda/2} = \left(1 + \frac{\lambda}{2}\right)$$

$$\therefore \theta = \theta_1\left(1 + \frac{\lambda}{2}\right) \longrightarrow ③$$

~~Eqn~~ Hence charge q becomes,

$$q = \frac{T}{2\pi} \cdot \frac{dI}{d\theta} \times \theta_1\left(1 + \frac{\lambda}{2}\right) \longrightarrow ④$$

Now to find λ,

$$\frac{\theta_1}{\theta_3} = e^{2\lambda}; \quad \frac{\theta_1}{\theta_5} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \frac{\theta_3}{\theta_4} \times \frac{\theta_4}{\theta_5} = d^{4\lambda}$$

$$\frac{\theta_1}{\theta_5} = d^{4\lambda}$$

$$\frac{\theta_n}{\theta_{n+1}} = e^{n\lambda}$$

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$\therefore \log_e \frac{\theta_1}{\theta_{n+1}} = n\lambda$

$\therefore 2.303 \log_{10} \frac{\theta_1}{\theta_{n+1}} = n\lambda$

$$\therefore \lambda = \frac{2.303}{n} \log_{10}\left(\frac{\theta_1}{\theta_{n+1}}\right) \longrightarrow (5)$$

OR, $\frac{\theta_1}{\theta_3} = e^{2\lambda}$ $\quad \therefore \left(\frac{\theta_1}{\theta_3}\right)^{1/4} = \left(e^{2\lambda}\right)^{1/4}$

$$\therefore \left(\frac{\theta_1}{\theta_3}\right)^{1/4} = e^{\lambda/2}$$

$$\therefore \left(\frac{\theta_1}{\theta_3}\right)^{1/4} = 1 + \frac{\lambda}{2} \longrightarrow (6)$$

Eqⁿ (4) becomes,

$$q = \frac{T}{2\pi} \cdot \frac{dI}{d\theta} \times \theta_1 \times \left(\frac{\theta_1}{\theta_3}\right)^{1/4} \longrightarrow (7)$$

This is most convenient formula for measuring charge q.

* Constants of Ballistic Galvanometer:-

(1) Figure of Merit:-

It is the current in microampere required to produce a deflection of 1 mm. on a scale. It is kept at a distance of one metre from mirror of galvanometer. Its unit is μA/mm.

If I_g is the current in microampere which produces a deflection θ mm on scale kept at a distance of one metre from mirror of galvanometer

$$\text{Figure of merit} = \frac{I_g}{\theta}$$

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(2) Current sensitivity :- (S)

It is deflection in millimeter produced by a current of one microampere on a scale kept at distance of 1 meter from mirror of galvanometer.

$$S = \frac{\theta}{I_g}$$

It is reciprocal of figure of merit.

Its unit is mm/μA.

(3) Voltage Sensitivity :-

It is the deflection in millimeter produced by potential difference of one microvolt across galvanometer on scale which kept at 1 metre from mirror of galvanometer.

$$\therefore \text{Voltage sensitivity} = \frac{\theta}{V_g} = \frac{\theta}{I_g . G}$$

$$= \frac{S}{G}$$

Its unit is mm/μV

(4) Charge Sensitivity :-

It is the corrected deflection in millimeter produced by a charge of 1 micro coulomb on a scale which kept at 1 metre from mirror of galvonometer.

Its unit is mm/μC.

If θ is corrected throw in mm by a charge q.

$$q = \frac{T}{2\pi} \cdot \frac{dI}{d\theta} \times \theta$$

$\therefore$ charge sensitivity $= \frac{\theta}{q} = \frac{2\pi}{T} \times \frac{d\theta}{dI}$

$$= \frac{2\pi}{T} \times \text{Current sensitivity.}$$

$\therefore$ Current sensitivity $= \frac{d\theta}{dI}$

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