Design of Experiments

by
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Basic terminologies:

Experiment: An experiment is a device to obtain answers to the problem under consideration. This can be classified into two categories:

a) ***Absolute experiments*** consist in determining the absolute value of some characteristics like i) finding the correlation coefficient of two variables ii) obtaining the average height of a group of people.
b) ***Comparative experiments*** are designed to compare the effect of two or more objects on some population characteristics, e.g., comparison of different fertilizers, different drugs, different medical experiments, different varieties of a crop etc.

Treatment: Various objects of comparison in a comparative experiment are called treatments. For example, in an agricultural experiment, different fertilizers or different varieties of crop are treatments.

Experimental unit: The smallest subdivision of the experimental material to which the treatments are applied and on which the variable under study is measured is called an experimental unit. Thus in an agricultural experiment the plot of land on which the treatment is applied is an experimental unit. In human experiments in which the treatment affects the individual, the individual will be the experimental unit.

Block: It is the group of homogeneous experimental units. In agricultural experiments, most of the times the whole experimental unit (field) is divided into relatively homogeneous sub-groups or strata. These strata which are more uniform among themselves than the field as a whole are known as blocks.

Replication: Replication means the execution of an experiment more than once. In other words the repetition of treatments under investigation is known as replication.

Precision: The precision (or, amount of information) of an experiment is measured by the reciprocal of the variance of a mean. Thus for an experiment replicated r times the precision is given by $1/Var(\bar{x}) = r/\sigma^2$, where σ^2 is the error variance per unit.

Efficiency of a Design: Consider the designs D_1 and D_2 with error variances per unit σ_1^2 and σ_2^2 and replications r_1 and r_2 respectively. Then the efficiency of design D_1 w.r.t. D_2 is given by the ratio:

Efficiency = Precision of D_1/ Precision of D_2 = $\frac{r_1}{\sigma_1^2} / \frac{r_2}{\sigma_2^2}$.

Experimental Error: Experimental Error is the random variation present in all experimental results. Different experimental units will give different responses to the same treatment, and it is often true that applying the same treatment over and over again to the same unit

will result in different responses in different trials. A part of this variation is systematic and can be explained, whereas the remainder is to be taken to be of random type. The unexplained random part of the variation is termed as the experimental error. This is not synonymous to mistake but technically it includes all types of extraneous variation due to:

i) The inherent variability in the experimental units
ii) Lack of uniformity in the methodology of conducting the experiment
iii) Lack of representativeness of the sample to the population under study.

Principles of Design:

For the validity of statistical analysis and enhancing the precision of the experiments three basic principals are observed according to R.A. Fisher who pioneered the study of experimental design. These are given by a) Randomization b) Replication and c) Local control.

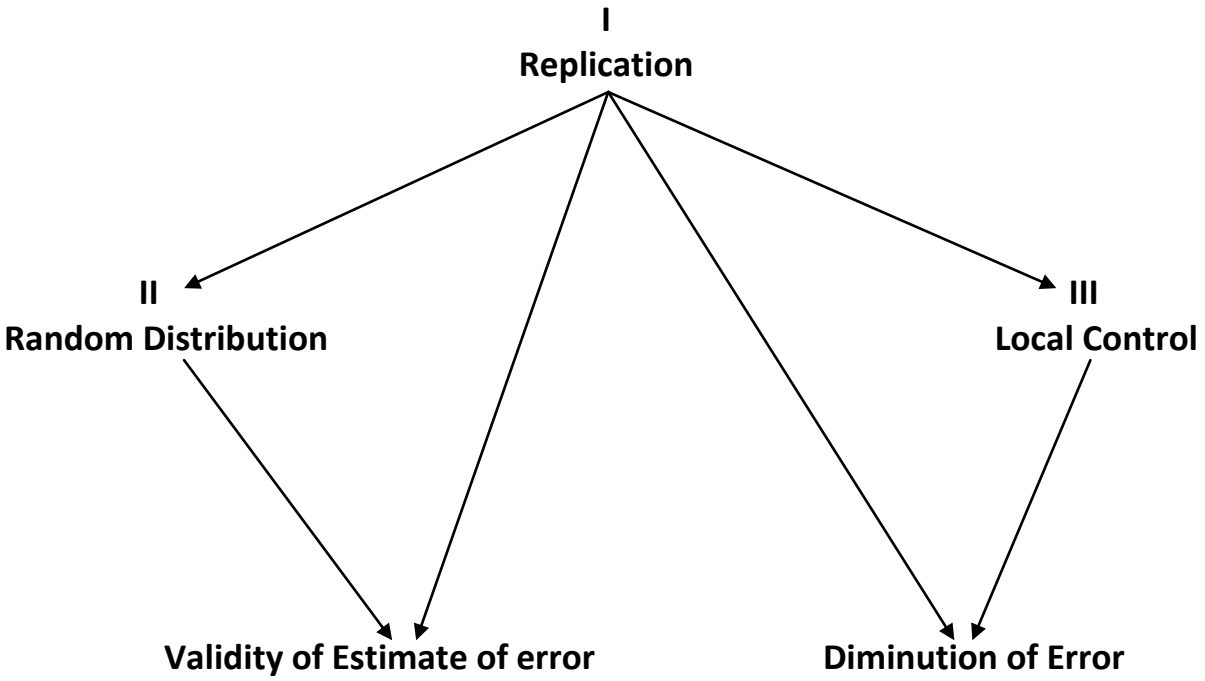
Replication: Replication means 'the repetition of the treatments under investigation'. In all experiments, some variation is introduced because of the fact that the experimental units such as individuals or plots of land in agricultural experiments cannot be physically identical. This type of variation can be removed by using a number of experimental units. We therefore perform the experiment more than once, i.e., we repeat the basic experiment. An individual repetition is called a replicate. The number, the shape and the size of replicates depend upon the nature of the experimental material. A replication is used:

i) to secure more accurate estimate of the experimental error
ii) to decrease the experimental error and thereby to increase precision, which is a measure of the variability of the experimental error
iii) to obtain more precise estimate of the mean effect of a treatment, since, $\sigma^2_{\bar{x}} = \frac{\sigma^2}{r}$ where r denotes the number of replications.

Randomization: This is a random process of assigning treatments to the experimental units. The random process implies that every possible allotment of treatments has the same probability. The purpose of randomization is to remove bias and other sources of extraneous variation, which are not controllable. Another advantage of randomization (accompanied by replication) is that it forms the basis of any valid statistical test. Hence the treatments must be assigned at random to the experimental units. Randomization leads to an unbiased estimate of variance as well as an unbiased estimate of treatment differences. It should be noted that randomization without replication is not sufficient. We are in the position for applying test of significance only when randomization is accompanied by an adequate no. of replications of treatments.

Local control: The process of reducing the experimental error by dividing the relatively heterogeneous experimental area (field) into homogeneous blocks is known as local control. In the field for agricultural experiment the soil heterogeneity plays an important role to increase the experimental error. It is desirable to reduce the experimental error as far as practicable without unduly increasing the no. of replications or without interfering with the statistical requirement of randomness. In order to separate soil fertility effects from the experimental error, the whole experimental area (field) is divided into homogeneous groups (blocks) row-wise or column-wise (One way elimination of fertility gradient) or both (elimination of fertility gradient in two perpendicular directions) according to the fertility

gradient of the soil such that the variation within each block is minimum and between the block is maximum. The treatments are then allocated at random within each block.
The following figure due to Fisher illustrates the functions of the various principals:



Choice of size and shape of plots and blocks:

Uniformity trials: The fertility of the soil does not increase or decrease uniformly in any direction but is distributed over the entire field in an erratic manner. *Uniformity trials* enable us to have an idea about the fertility variation of the field. By uniformity trial, we mean a trial in which the field (experimental material) is divided into small units (plots) and the same treatment is applied on each of the units and the yields are recorded. From these yields, we can draw a 'fertility contour map' which gives us a graphic picture of the variation of the soil fertility. The fertility contour map is obtained by joining the points of equal fertility through lines. Uniformity trials also give some idea about the shape and size of the plots to be used. From the fertility contour map, it is generally observed that adjacent plots are more or less alike in fertility than those apart. Thus a homogeneous block can be formed by combining a number of adjacent plots.
Size of the Plot: The size of the plot depends on a no. of factors such as the total experimental area available, the no. of treatments, the no. of replications of each treatment, the crop and so on. The total available experimental area remaining fixed, an increase in the size of the plots will automatically decrease the no. of plots and indirectly increase the block size while reducing the no. of blocks. Also, as the no. of plots increases, the no. of guard areas and hence the amount of non-experimental area also increases. These facts should be kept in view while deciding on the size of plots.

An important investigation on the effect of size and shape of plot and block was conducted by H. Fairfield Smith. After conducting uniformity trial experiments with the same crop and then harvesting the crop in small units, he obtained an empirical relationship between the plot size and the plot variance. This relationship, known as Fairfield Smith's variance law is given by,

$V_x = V_1 / x^b$

$\Rightarrow \log V_x = \log V_1 - b \log x$

Where, V_x → *variance of the yield per unit area from plot of size x units*

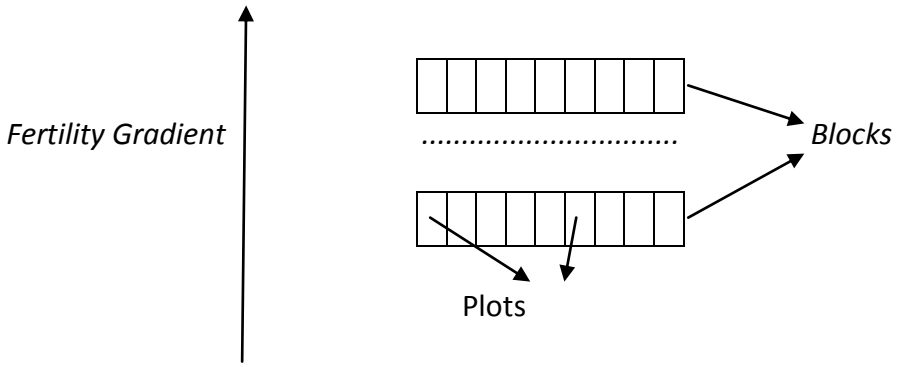
b → *soil characteristics*, $0 <= b <= 1$

$b = 1$ means that the units making the plot of size x units are not correlated and then $V_x = V_1 / x$, so that an increase in plot size increases the precision of the experiment.

$b = 0$ means that the units of the plot of size x units are perfectly correlated and then $V_x = V_1$, so that there is no gain in precision by increasing plot size.

Usually, $0 < b < 1$ and an increase in plot size increases the precision of the experiment.

Shape of Blocks and Plots: The shape and size of the blocks will usually depend upon the shape and size of the plots. When definite fertility contours are present the maximum precision will be obtained by arranging the plots in a block with their long sides parallel to the direction of the fertility gradient.



In the absence of any knowledge about fertility contours, it is better to use square plots and generally, it is best to have small blocks. Otherwise the plots within a block will not be homogeneous.

Advantages and disadvantages of CRD:

1. This design is most commonly used in laboratory experiments such as in Agricultural Chemistry, plant pathology, and animal experiments where the experimental material is expected to be homogeneous.

2. This design is useful in pot cultural experiments where the same type of soil is usually used. However, in greenhouse experiments care has to be taken with regard to sunshade, accessibility of air along and across the bench before conducting the experiment.

3. Any number of replications and treatments can be used. The number of replications may vary from treatment to treatment.

4. The analysis remains simple even if information on some units is missing.

5. This design provides maximum number of degrees of freedom for the estimation of error than the other designs.

6. The only drawback with this design is that when the experimental material is heterogeneous, the experimental error would be inflated and consequently the treatments are less precisely compared. The only way to keep the experimental error under control is to increase the number of replications thereby increasing the degrees of freedom for error.

Applications: 1. CRD is most useful in laboratory technique and methodological studies.

Ex: in physics, chemistry, in chemical and biological experiments, in some greenhouse studies etc.

2. CRD is also recommended in situations where an appreciable fraction of units is likely to be destroyed or fail to respond.

Advantages and disadvantages of RBD:

1. The principle advantage of RBD is that it increases the precision of the experiment. This is due to the reduction of experimental error by adoption of local control.

2. The amount of information obtained in RBD is more as compared to CRD. Hence, RBD is more efficient than CRD.

3. Flexibility is another advantage of RBD. Any number of replications can be included in RBD. If large numbers of homogeneous units are available, large number of treatments can be included in this design.

4. Since the layout of RBD involves equal replication of treatments, statistical analysis is simple. Even when some observations are missing of certain treatments, the data can be analysed by the use of missing plot technique.

5. When the number of treatments is increased, the block size will increase. If the block size is large it may be difficult to maintain homogeneity within blocks. Consequently, the experimental error will be increased. Hence, RBD may not be suitable for large number of treatments. But for this disadvantage, the RBD is a versatile design. It is the most frequently used design in agricultural experiments.

Advantages and disadvantages of Latin Square Design (L.S.D.):

1) With two way grouping or stratification LSD controls more of the variation than C.R.D. or R.B.D.

2) L.S.D. is an incomplete 3-way layout. Its advantage over complete 3-way layout is that instead of m^3 experimental units only m^2 units are needed. Thus a 4x4 L.S.D. results in saving of 64-16 = 48 observations over a complete 3-way layout.

3) The statistical analysis is simple though slightly complicated than for R.B.D. Even with missing data the analysis remains relatively simple.

4) More than one factor can be investigated simultaneously.

5) The missing observations can be analysed by using missing plot technique.

6) A serious limitation of the L.S.D. is that the no. Of replicates must be the same as the no. of treatments. As a result, square larger than 12X12 are seldom used, as the square does not remain homogeneous for its large size. On the other hand, small squares provide only a few degrees of freedom for the error. The most commonly used sizes are 5X5 to 8X8.

7) Another disadvantage is that the analysis depends heavily on the assumption that there are no interactions present.

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