03/08/2023.

Page No.
Date

Superpositions of Harmonic Oscillations.

Introduction :-

we know that, SHM is represented by a homogeneous linear differential eqn,

$$\frac{d^2y}{dt^2} = -\omega^2 y.$$

$F \propto -y$

$\therefore$ $F \propto$ displacement from mean position.

$\therefore$ $F \propto -$displacement (y)

$\therefore$ homogeneous linear equations having property is sum of any two solutions is also a solution.

$\therefore$ when partcle subjected two simple harmonic oscillations simultaneously.

Resultant displacement $y = y_1 + y_2$

y_1 and y_2 are displacements produced by individual simple harmonic motions.

principle of superposition which is possible only in case of homogeneous linear equations.

* Resultant motion of the particle traces a curve called Lissajous figure.
* The ~~sph~~ shape of Lissajous figure depends on the frequency, amplitude, and phase differenc of the two constituent simple harmonic oscillations.

* Linearity and superposition principle.

A SHM is generally represented by a differential equation given by.

$$\frac{d^2y}{dt^2} = -\omega^2 y \quad \text{--- (1)}$$

F (restoring force) is depends only on displacement (y)

equation does not contain higher term called linear equations.
equation contains higher order term called non-linear equations.
* homogenous linear equation :- having property sum of two solutions having solutions.

* Principle of superpositions :-
statement :- It state that, when a particle is subjected to two simple harmonic motions simultaneously, then the Resultant displacement ($\bar{y}$) is the vector sum of the individual displacement that they can produce $y = \bar{y}_1 + \bar{y}_2$.

only homogenous linear equations obey the principle of superposition.

Let us consider a non-homogeneous, non linear differential equations.

$$\frac{d^2y}{dt^2} = A - \omega^2 y + By^2 + Cy^3 + Dy^4 + \cdots \quad \text{—— (1)}$$

If y_1 and y_2 are two different solutions ~~then~~ of diff. equations (1). then

$$\frac{d^2y_1}{dt^2} = A - \omega^2 y_1 + By_1^2 + Cy_1^3 + Dy_1^4 + \cdots \quad \text{—— (2)}$$

and $$\frac{d^2y_2}{dt^2} = A - \omega^2 y_2 + By_2^2 + Cy_2^3 + Dy_2^4 \cdots \quad \text{—— (3)}$$

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If principle of superposition is true, then,

$$y = y_1 + y_2$$

$$\therefore \frac{d^2y}{dt^2} = \frac{d^2(y_1+y_2)}{dt^2} = A - \omega^2(y_1+y_2) + B(y_1+y_2)^2 + C(y_1+y_2)^3 + \cdots \quad \text{——— (4)}$$

Now adding eqn (2) and (3) we get,

$$\therefore \frac{d^2y_1}{dt^2} + \frac{d^2y_2}{dt^2}$$

$$= A - \omega^2 y_1 + By_1^2 + Cy_1^3 + Dy_1^4 + \cdots$$

$$+ \; A - \omega^2 y_2 + By_2^2 + Cy_2^3 + Dy_2^4 + \cdots$$

$$= 2A - \omega^2(y_1+y_2) + B(y_1^2+y_2^2) + C(y_1^3+y_2^3) + D(y_1^4+y_2^4) + \cdots \quad \text{——— (5)}$$

eqn (4) and (5) are identical only if,

$$\frac{d^2(y_1+y_2)}{dt^2} = \frac{d^2y_1}{dt^2} + \frac{d^2y_2}{dt^2}$$

if $A = 2A$.

$$\therefore -\omega^2(y_1+y_2) = -\omega^2 y_1 - \omega^2 y_2 \quad \text{——— (6)}$$

$$\therefore B(y_1+y_2)^2 = B(y_1^2+y_2^2) \quad \text{——— (7)}$$

$$\therefore C(y_1+y_2)^3 = C(y_1^3+y_2^3) \quad \text{——— (8)}$$

if A, B, C are identically zero then equation are true.

only homogenous linear eqn obey principle of superposition.

* Superposition of two Collinear Harmonic Oscillations Having Equal Frequencies

i) Analytical Method :-

Consider two simple harmonic motions with same frequency (ω) and different amplitudes (a_1, a_2) and different initial phases (α, α_2) as,

$$y_1 = a_1 \sin(\omega t + \alpha_1)$$

and $y_2 = a_2 \sin(\omega t + \alpha_2)$

$\because$ According to the principle of superposition,

$\therefore$ $y = y_1 + y_2$

$\therefore y = a_1 \sin(\omega t + \alpha_1) + a_2 \sin(\omega t + \alpha_2)$

by using formula $\sin(A+B)$

$$y = a_1[\sin\omega t \cdot \cos\alpha_1 + \cos\omega t \cdot \sin\alpha_1] + a_2[\sin\omega t \cdot \cos\alpha_2 + \cos\omega t \sin\alpha_2]$$

~~$y = a_1 \cos\alpha_1 +$~~

$$y = a_1 \sin\omega t \cdot \cos\alpha_1 + a_1 \cos\omega t \cdot \sin\alpha_1 + a_2 \sin\omega t \cdot \cos\alpha_2 + a_2 \cos\omega t \cdot \sin\alpha_2.$$

$$y = \sin\omega t\,[a_1 \cos\alpha_1 + a_2 \cos\alpha_2] + \cos\omega t\,[a_1 \sin\alpha_1 + a_2 \sin\alpha_2] \quad \text{--- (1)}$$

a_1, a_2 and α_1, α_2 are constants.

we can put,

$$\therefore A\cos\phi = a_1 \cos\alpha_1 + a_2 \cos\alpha_2 \quad \text{--- (2)}$$

$$\therefore A\sin\phi = a_1 \sin\alpha_1 + a_2 \sin\alpha_2 \quad \text{--- (3)}$$

Page No.
Date

Now, squaring and adding eqn ② and ③

$\therefore A^2(\sin^2\phi + \cos^2\phi) = (a_1\sin\alpha_1 + a_2\sin\alpha_2)^2 + (a_1\cos\alpha_1 + a_2\cos\alpha_2)^2$

$= a_1^2\sin^2\alpha_1 + a_2^2\sin^2\alpha_2 + 2a_1\sin\alpha_1 \cdot a_2\sin\alpha_2 + a_1^2\cos^2\alpha_1 +$
$a_2^2\cos^2\alpha_2 + 2a_1\cos\alpha_1 \cdot a_2\cos\alpha_2$

$= a_1^2(\sin^2\alpha_1 + \cos^2\alpha_1) + a_2^2(\sin^2\alpha_2 + \cos^2\alpha_2) +$
$2a_1a_2(\sin\alpha_1 \cdot \sin\alpha_2 + \cos\alpha_1\cos\alpha_2)$

$\therefore A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos(\alpha_1 - \alpha_2)$ — ④

and taking the ratio of eqn ② & ③

$$\therefore \frac{A\sin\phi}{A\cos\phi} = \frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2}$$

$$\therefore \tan\phi = \frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2}$$

in terms of A & ϕ the eqn Ⓘ becomes

$y = A\cos\phi \cdot \sin\omega t + A\sin\phi \cdot \cos\omega t$.
$y = A\sin(\omega t + \phi)$

$\therefore \boxed{y = A\sin(\omega t + \phi)}$

this eqn represent the resultant SHM.

As a special case, if $\alpha_1 = \alpha_2 = \alpha$ then,

$A = a_1 + a_2$

& $\phi = \alpha$.

Page No.
Date

* Superpositions of Two collinear Harmonic oscillations having Different Frequencies (Beats)

If two sound sources of nearly same frequency and amplitude are sounded together, then at any point the phase difference between two wave meeting goes on changing continuously.

wave meets in phase when maximum sound is produced and wave meet out of phase when minimum sound is produced. Thus maxima and minima of sound are produced which is called waxeing and waning of sound and are known as beats.

maxima → waxeing
minima → wanning.

* Analytical Treatment of Beats.

Consider two sound sources with frequencies n_1 & n_2
Let, a and b be the amplitudes of the waves resp.
Let us suppose that at time $t=0$. two waves are in phase
displacement of y_1 and y_2.

$$\therefore \quad y_1 = a \sin \omega_1 t = a \sin 2\pi n_1 t$$

$$\& \quad y_2 = b \sin \omega_2 t = b \sin 2\pi n_2 t .$$

The resultant displacement $y = y_1 + y_2$.
According to superposition principle is $y = y_1 + y_2$.

$$y = a \sin 2\pi n_1 t + b \sin 2\pi n_2 t .$$

$$y = a \sin 2\pi n_1 t + b \sin 2\pi [n_1 - (n_1 - n_2)] t .$$

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$\sin(A-B)$
$= \sin A \cos B - \cos A \sin B$

$y = a \sin 2\pi n_1 t + b[\sin 2\pi n_1 t \cdot \cos 2\pi (n_1 - n_2)t$
$- \cos 2\pi n_1 t \cdot \sin 2\pi (n_1 - n_2)t]$

$= \sin 2\pi n_1 t \, [a + b \cos 2\pi (n_1 - n_2)t] -$
$\cos(2\pi n_1 t) \cdot b \sin 2\pi (n_1 - n_2)t \,]$

Now, put, $a + b \cos 2\pi (n_1 - n_2)t = A \cos\theta$
$b \sin 2\pi (n_1 - n_2)t = A \sin\theta$

$\therefore A^2 \cos^2\theta + A^2 \sin^2\theta = [a + b \cos 2\pi (n_1 - n_2)t]^2 +$
$[b \sin 2\pi (n_1 - n_2)t]^2$

$\therefore A^2 = [a^2 + b^2 \cos^2 2\pi (n_1 - n_2)t + 2ab \cos 2\pi (n_1 - n_2)t]$
$+ b^2 \sin^2 2\pi (n_1 - n_2)t$

$\therefore A^2 = a^2 + b^2 + 2ab \cos 2\pi (n_1 - n_2)t$ —— ①

$\therefore A = \sqrt{a^2 + b^2 + 2ab \cos 2\pi (n_1 - n_2)t}$

and

~~$\tan\theta$~~ =

$$\frac{A \sin\theta}{A \cos\theta} = \frac{b \sin 2\pi (n_1 - n_2)t}{a + b \cos 2\pi (n_1 - n_2)t} \quad —— ②$$

$$\therefore \tan\theta = \frac{b \sin 2\pi (n_1 - n_2)t}{a + b \cos 2\pi (n_1 - n_2)t}$$

$\therefore y = A \sin 2\pi n_1 t \cdot \cos\theta - A \cos 2\pi n_1 t \cdot \sin\theta$
$y = A \sin(2\pi n_1 t - \theta)$ —— ③

is resultant displacement with amplitude

$A = \sqrt{a^2 + b^2 + 2ab \cos 2\pi (n_1 - n_2)t}$

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Page No.
Date

(i) Maxima :- For maximum sound the amplitude must be maximum at a certain time t, which requires the condition,

$$\therefore 2\pi(n_1-n_2)t = 2\pi k \quad \text{where } k = 0,1,2,3 \dots$$

ie. at time $t = \frac{k}{n_1-n_2}$

at instances $0, \frac{1}{(n_1-n_2)}, \frac{2}{(n_1-n_2)}, \frac{3}{(n_1-n_2)}, \dots$

amplitude and loudness (sound) is maximum.

$$\therefore \text{Beat period } T = \frac{1}{(n_1-n_2)}$$

$$\therefore \text{Beat frequency } N = (n_1-n_2).$$

ii) Minima :- amplitude (A) must be minimum which requires the condition that,

$$2\pi(n_1-n_2t) = (2k+1)\pi \quad \text{where, } k = 0,1,2 \dots$$

ie. at time $t = \frac{2k+1}{2(n_1-n_2)}$

at time instances $\frac{1}{2(n_1-n_2)}, \frac{3}{2(n_1-n_2)}, \frac{5}{2(n_1-n_2)}, \dots$

$$\therefore \text{Beat period} = T = \frac{1}{(n_1-n_2)}$$

$$\therefore \text{Beat frequency } N = \frac{1}{T} = (n_1-n_2)$$

if $a = b$ then the minimum amplitude become zero and hence maxima & minima [illegible] distinct.

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* Superposition of two perpendicular Harmonic oscillations Having Equal Frequencies.

ⓐ Analytical method :-

Let, $x = a \sin(\omega t + \alpha)$ ——— 25 ①

& $y = b \sin \omega t$ ——— ②

be the displacements produced by two SHM's acting simultaneously i.e. one along x-axis & another along y-axis

from eq^n ②

$\sin\omega t = \dfrac{y}{b}$ $\therefore$ $\sin^2\omega t = \dfrac{y^2}{b^2}$.

and $\cos\omega t = \sqrt{1 - \dfrac{y^2}{b^2}}$

from eq^n ①

$$\frac{x}{a} = \sin\omega t \cdot \cos\alpha + \cos\omega t \cdot \sin\alpha$$

$$\frac{x}{a} = \left[\frac{y}{b} \cdot \cos\alpha + \sqrt{1-\frac{y^2}{b^2}} \cdot \sin\alpha\right]$$

$$\therefore \frac{x}{a} - \frac{y}{b}\cos\alpha = \sqrt{1-\frac{y^2}{b^2}} \cdot \sin\alpha$$

$\therefore$ squaring this eq^n, we get,

$$\therefore \left(\frac{x}{a} - \frac{y}{b}\cos\alpha\right)^2 = \left(\sqrt{1-\frac{y^2}{b^2}}\right)^2 \sin^2\alpha$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\alpha - 2\frac{x}{a}\cdot\frac{y}{b}\cdot\cos\alpha = \left(1-\frac{y^2}{b^2}\right)\sin^2\alpha$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\alpha + \frac{y^2}{b^2}\sin^2\alpha - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha$$

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$$\therefore \cancel{\frac{x^2}{b}} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2}(\cos^2\alpha + \sin^2\alpha) - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha.$$

$$\therefore \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha.} \quad \text{——— (3)}.$$

This is equation for an ellipse, which represents the locus of the particle subjected two SHM's at right angles to each other simultaneously.

Depending upon the phase difference (α) we have different shapes of the locus of the particles called Lissajous figures.

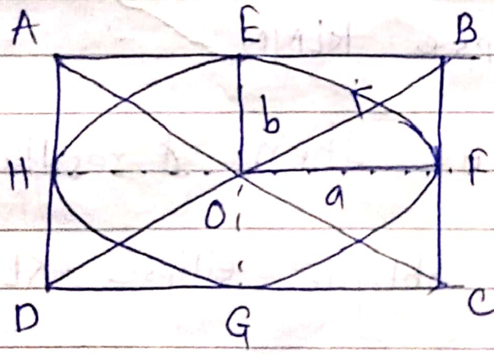
i) if $\alpha = 0$ or 2π then $\sin\alpha = 0$ & $\cos\alpha = 1$

$\therefore$ from eqn (3) we get,

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0. \quad \text{or} \quad \left(\frac{x}{a} - \frac{y}{b}\right) = 0.$$

or $\frac{x}{a} = \frac{y}{b}$

$\therefore \boxed{y = \frac{b}{a} \cdot x.}$ is an equation for a straight line DB.



[Lissajous figures]

if

ii) if $\alpha = \pi$ then $\sin x = 0$ and $\cos x = -1$

$\therefore$ from eqn ③ $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$

$$\therefore \left(\frac{x}{a} + \frac{y}{b}\right) = 0$$

$$\therefore \frac{x}{a} = -\frac{y}{b}$$

$\therefore \boxed{y = -\frac{b}{a}x}$ represents a straight line AC

iii) if $\alpha = \frac{\pi}{2}$, or $\frac{3\pi}{2}$ then $\sin \alpha = 1$ & $\cos \alpha = 0$.

$\therefore$ eqn ③ becames $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

This represents an ellipse EHGF with semi major axis (a) and semiminor axis (b)

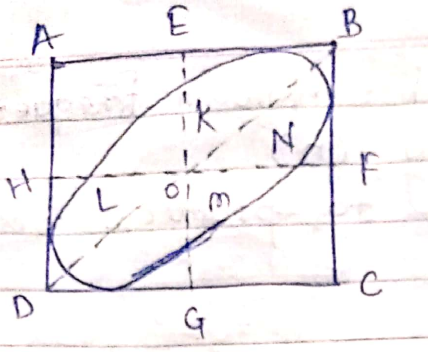
iv) If $\alpha = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ & $a = b$ then eqn ③ becames,

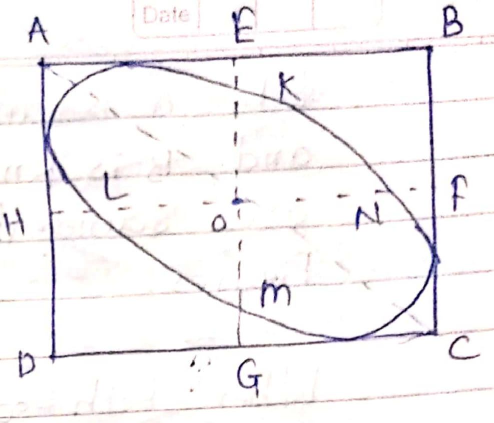
$$x^2 + y^2 = a^2.$$

this represent a circle with radius (a).

v) If $\alpha = \frac{\pi}{4}$ or $\frac{7\pi}{4}$, then vibration is along an oblique ellipse KLMN.

& if $\alpha = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$ then & resultant vibration is along an oblique ellipse KLMN





Uses of Lissajous figures :-

(a) To determine the ratio of time periods or frequencies :-
from the observation of Lissajous figures, The number of times the curve touches the horizontal and vertical sides of a rectangle bounding the Lissajous figure is found.

1) If the curve touches horizontal side m times and vertical side n time, then the ratio of time period is m:n or the ratio of frequencies is n:m.
horizantal side once & vertical side twice The ratio of ~~frequencies is~~ & periods is 1:2.

2) The ratio of frequencies is only measured when the ratio is simple integral numbers.

When ~~frequcecies~~ frequencies are nearly equal:-

① Unknown frequency of a tuning fork can be determined if the frequency of the other tuning fork is known.

Let, A is tuning Fork with unknown frequency (n_1) and B is known frequency (n) are sounded together to produce Lissajous Figures.

When both sources start vibrating in phase the resultant ~~be~~ will be straight line. As time passes difference goes on increasing and hence the shape of resultant curve goes on changing i.e. different Lissajous Figures are traced.

$$\therefore \text{ frequency } p = \frac{1}{t}$$

$$\therefore n_1 - n = p$$

$$\therefore n_1 = n \pm p$$

Now, fix little wax on the prong of tuning fork A and repeat observations and find the diff. in frequency as.

$$p' = \frac{1}{t'}$$

i) if $t' < t$

then $p' < p$ is the observation and if

if $n_1 > n$ then $n_1 - n = p$ and $n_1' - n = p'$

for a loaded tuning fork frequency decreases

$n_1' < n_1$

$\therefore p' < p$ which contradicts the observation ($p' > p$)

$\therefore n_1 - n = p$ is wrong

$\therefore$ Hence, $n > n_1$

$\therefore n - n_1 = p$ or $n_1 = n - p$ is the unknown frequency.

ii) If $t' > t$ then $p' < p$ is the observation and if $n_1 > n$ then $n_1 - n = p$.
and $n_1' - n = p'$ for loaded A.
since $n_1' < n_1$ we see that $p' < p$ which justifies the observation.
$\therefore n_1 - n = p$.
Or $n_1 = n + p$ is unknown frequency of (A).

* Superposition of two SHM's at Right angles to each other and having frequencies in the ratio 2:1.

i) Analytical method :-
Let us consider two SHM's one along x-axis and another along y-axis with amplitudes a and b resp. and diff. in phase by α. frequencies are in the ratio 2:1. so the analytical eqn for two SHM's are,

$$x = a \sin(2\omega t + \alpha) \quad \text{——} (1)$$

$$\text{and } y = b \sin(\omega t) \quad \text{——} (2)$$

$$\therefore \frac{y}{b} = \sin(\omega t) \qquad \therefore \frac{y^2}{b^2} = \sin^2 \omega t.$$

$$\text{and} \quad \cos \omega t = \sqrt{1 - \frac{y^2}{b^2}}$$

$$\therefore \frac{x}{a} = \sin(2\omega t + \alpha)$$
$$= \sin 2\omega t \cdot \cos\alpha + \cos 2\omega t \cdot \sin\alpha$$

$$\frac{x}{a} = 2 \sin\omega t \cdot \cos\omega t \cdot \cos\alpha + (1 - 2\sin^2\omega t) \sin\alpha \quad \text{——} (3)$$

$$\therefore \left(\cos 2\omega t = \cos^2\omega t - \sin^2\omega t \right.$$
$$\left. = 1 - \sin^2\omega t - \sin^2\omega t \right)$$

Now, substituting for $\sin\omega t$ and $\cos\omega t$ in eqⁿ (3) we get

$$\frac{x}{a} = 2\cdot\frac{y}{b}\sqrt{1-\frac{y^2}{b^2}}\cdot\cos\alpha + \left(1-\frac{2y^2}{b^2}\right)\sin\alpha.$$

$$\therefore \frac{x}{a} - \left(1-\frac{2y^2}{b^2}\right)\cdot\sin\alpha = 2\frac{y}{b}\sqrt{1-\frac{y^2}{b^2}}\cdot\cos\alpha$$

$$\therefore \left[\left(\frac{x}{a}-\sin\alpha\right)+\frac{2y^2}{b^2}\sin\alpha\right] = \frac{2y\cos\alpha}{b}\sqrt{1-\frac{y^2}{b^2}}$$

squaring both the sides of above eqn.

$$\therefore \left(\frac{x}{a}-\sin\alpha\right)^2 + \frac{4y^4}{b^4}\sin^2\alpha + 2\left(\frac{x}{a}-\sin\alpha\right)\frac{2y^2}{b^2}\sin\alpha = \frac{4y^2\cos^2\alpha}{b^2}\left(1-\frac{y^2}{b^2}\right)$$

$$\therefore \left(\frac{x}{a}-\sin\alpha\right)^2 + \frac{4y^4}{b^4}\sin^2\alpha + \frac{4y^2}{b^2}\cdot\frac{x}{a}\cdot\sin\alpha - \frac{4y^2}{b^2}\sin^2\alpha = \frac{4y^2\cos^2\alpha}{b^2}\left(1-\frac{y^2}{b^2}\right)$$

$$\therefore \left(\frac{x}{a}-\sin\alpha\right)^2 + \frac{4y^4}{b^4}\sin^2\alpha + \frac{4y^2}{b^2}\frac{x}{a}\sin\alpha - \frac{4y^2}{b^2}\sin^2\alpha = \frac{4y^2}{b^2}\cos^2\alpha - \frac{4y^4}{b^4}\cos^2\alpha$$

$$\therefore \left(\frac{x}{a}-\sin\alpha\right)^2 + \frac{4y^4}{b^4}(\sin^2\alpha+\cos^2\alpha) + \frac{4y^2}{b^2}\frac{x}{a}\sin\alpha = \frac{4y^2}{b^2}(\sin^2\alpha+\cos^2\alpha)$$

$$\therefore \left(\frac{x}{a}-\sin\alpha\right)^2 + \frac{4y^4}{b^4} + \frac{4y^2}{b^2}\frac{x}{a}\sin\alpha - \frac{4y^2}{b^2} = 0$$

$$\therefore \left(\frac{x}{a} - \sin\alpha\right)^2 + \frac{4y^2}{b^2}\left[\frac{y^2}{b^2} + \frac{x}{a}\sin\alpha - 1\right] = 0 \quad \text{--- (4)}$$

This equation represend the general eqn of curve having two loops.

(a) If $\alpha = 0, \pi, 2\pi$ etc. the $\sin\alpha = 0$.
eqn (4) becames,

$$\therefore \frac{x^2}{a^2} + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} - 1\right) = 0.$$

This eqn represent two loops looking like 8.

b) If $\alpha = \frac{\pi}{2}$ then $\sin\alpha = 1$
eqn (4)

$$\left(\frac{x}{a} - 1\right)^2 + \frac{4y^2}{b^2}\left[\frac{y^2}{b^2} + \frac{x}{a} - 1\right] = 0$$

$$\therefore \left(\frac{x}{a} - 1\right)^2 + \frac{4y^2}{b^2}\left(\frac{x}{a} - 1\right) + \frac{4y^4}{b^4} = 0$$

$$\therefore \left[\left(\frac{x}{a} - 1\right) + \frac{2y^2}{b^2}\right]^2 = 0.$$

$$\therefore \frac{2y^2}{b^2} = -\left(\frac{x}{a} - 1\right)$$

$$\therefore y^2 = -\frac{b^2}{2a}(x - a)$$

$\therefore$ This eqn shows a parabola, with vertex at $(a, 0)$.

For different values of phase difference (α) are given by different Lissajous Fig.

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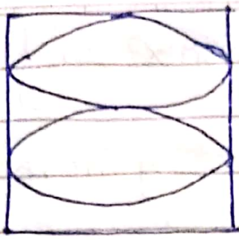
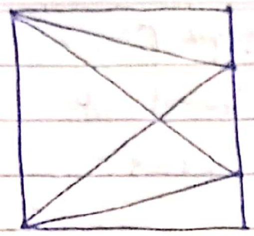
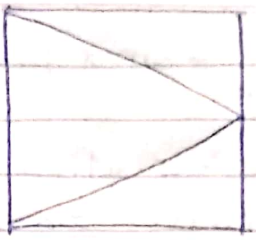


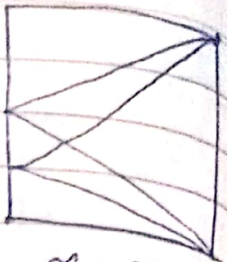
$\alpha = 0$



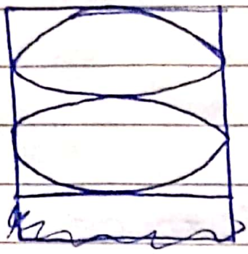
$\alpha = \pi/4$



$\alpha = \pi/2$.



$\alpha = \frac{3\pi}{4}$



$\alpha = \pi$.