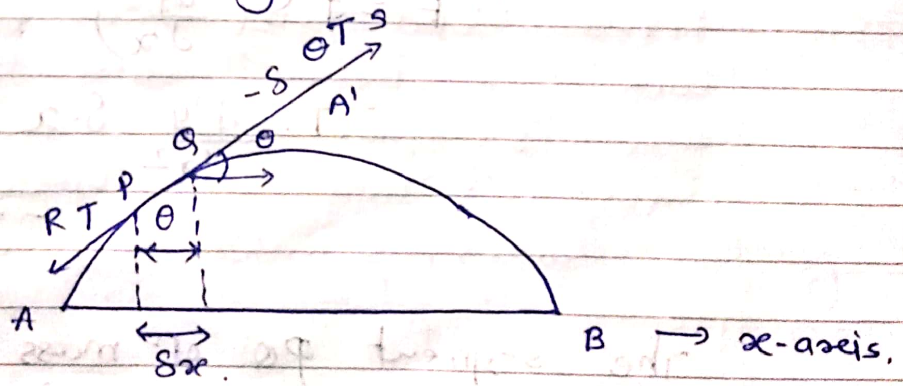
3. Wave Motion and Ultrasonic Waves.

Introduction :- We are going to study transverse vibrations of stretched strings.
If the length and tension are properly adjusted for a given string the musical notes can be created. The string can vibrate in different modes which are standing waves produced by the superposition of identical waves travelling in opposite directions due to reflection from fixed end points.
Ultrasonic waves are high frequency sound waves beyond audible limit (ie. > 20,000 Hz)
They can be produced using piezoelectric generator. The same technique can be used to detect the ultrasonic waves.
uses → industry, medical science, sound signalling and measurement of sea ~~dp~~ depth. etc.

* Transverse waves on a string -
* Expression for velocity of transverse waves on a string.



* Motion of a stretched string *

Consider a Flexible string AB stretched under a tension T and fixed at point A and B. ~~Let m~~ Let, m be the mass per unit length of the string.
length AB of the string is along x-axis and string is displaced along y-axis through a small distance.

Now consider a small segment PQ of length δx in displaced position of the string.
Resolving the tensions at P and Q along horizontal and vertical components.
$\therefore$ restoring force $\vec{F}$ acts on PQ in y- direction

$$F = T\sin\theta - T\sin(\theta - \delta\theta)$$
$$= T[\sin\theta - \sin\theta \cdot \cos\delta\theta + \cos\theta \cdot \sin\delta\theta]$$
$$\simeq T\cos\theta \cdot \delta\theta \quad \text{---} (\because \cos\theta \approx 1,\ \sin\delta\theta =$$
$$= T\,\delta(\sin\theta)$$
$$= T\,\delta(\tan\theta) \quad \text{---} (\because \sin\theta = \tan\theta$$

$$\therefore F = T\,\delta\left(\frac{dy}{dx}\right)$$
$$= T \cdot \frac{d^2y}{dx^2}\,\delta x \quad \text{———} \quad (1)$$
$$\text{---} \left(\because \delta\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right)\right.$$

The segment PQ of mass $(m \cdot \delta x)$ is subjected to a restoring force $\vec{F}$ and ~~d~~ acceleration $\frac{d^2y}{dt^2}$ in y- direction.
$\therefore$ the equation of motion for the segment PQ is,

$$(m \cdot \delta x)\frac{d^2y}{dt^2} = T \cdot \frac{d^2y}{dx^2} \cdot \delta x.$$

$$\therefore \frac{d^2y}{dt^2} = \frac{T}{m} \cdot \frac{d^2y}{dx^2}.$$

$\therefore$ comparing this diff. equation. of motion for a vibration string.

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

we get, an expression for the velocity of wave travelling along a stretched string as,

$$v^2 = \frac{T}{m} \quad \text{or} \quad \boxed{v = \sqrt{\frac{T}{m}}}$$

* Travelling and standing waves on a string.

i) Travelling waves :- A flexible string fixed at end points when stretched under tension (T) shows elastic properties, Due to tension in the string restoring force are created.
Due to elasticity in the string, the disturbance created at a point moves along the string with a velocity $v = \sqrt{\frac{T}{m}}$.

where, m is mass per unit length of the string
This wave is called travelling wave.
The displacement (y) of the particle at any point at a distance (x) from the source of disturbance on the string at time (t) is expressed as,

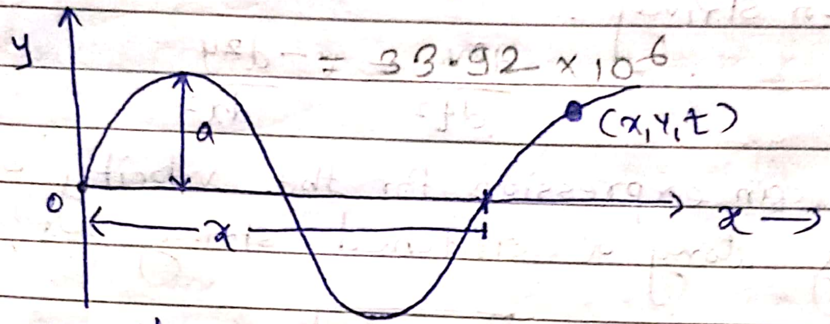
$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

$y = a \sin(\omega t - kx)$

where, $\frac{2\pi}{\lambda} = k$

& $\omega = 2\pi A$

$= 2\pi \frac{v}{\lambda}$



All the particles perform SHM with same frequency and amplitude. At a given time the phase of successive particles are different. The particle velocity $\left(\frac{dy}{dt}\right)$ is different from wave velocity.

* **Standing waves:—** Waves when reach the fixed end points ~~the~~ ~~vo~~ waves get reflected back and hence start travelling in opposite direction. Thus there are two progressive waves travelling in opposite direction. The superposition of two waves results in producing standing or stationary waves.

Two progressive waves travelling in opposite directions, but with same velocity, frequency, wavelength and amplitude may be represented as,

$$y_1 = a \sin(\omega t - kx) \quad \text{——— (1) Direct.}$$

&

$$y_2 = -a \sin(\omega t + kx) \quad \text{——— Reflected.}$$

negative sign indicate phase reversal at the reflection from fixed point.

$$y = y_1 + y_2 = a[\sin(\omega t - kx) - \sin(\omega t + kx)]$$

$$= [2a \sin(kx)] \cdot \cos\omega t.$$

$$= A \cos(\omega t). \quad \text{——— (2)}$$

where, $A = 2a \sin(kx)$ is amplitude of the particle.

from eqn (2) it is clear that the particles of the medium perform SHM with the same frequency (ω) but they have diff. amplitudes (A) which is a function of x. The fixed end points are always at rest. ie. their amplitudes are zero.

amplitude is zero called nodes and also there are points where, amplitude is maximum called antinodes.

a) Nodes :- The nodes occur at distance x, given by,

$$A = 2a \sin kx = 0$$

$$\therefore kx = m\pi$$

$$\therefore \frac{2\pi}{\lambda} x = m\pi$$

where, $m = 0, 1,$

Nodes occure at distance $x = m \cdot \frac{\lambda}{2}$.

$$= 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$$

Thus successive nodes are separated by a distance $\frac{\lambda}{2}$.

(b) Antinodes :- The antinodes correspond to posit where, the amplitude is maximum. The condition for antinodes is given by,

$$A = 2a \sin kx = \pm 1$$

$$\therefore kx = (2m+1)\frac{\pi}{2} \qquad \text{where, } m = 0, 1, 2 \ldots$$

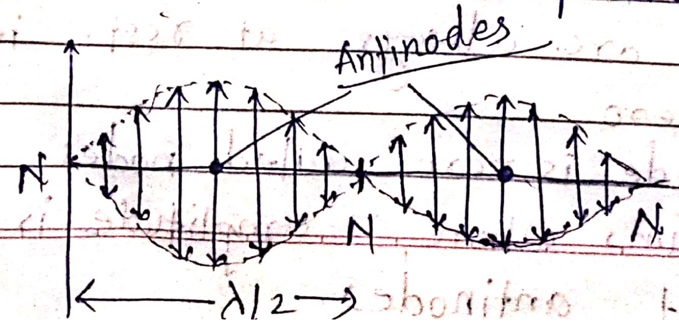
$$\therefore \frac{2\pi}{\lambda} \cdot x = (2m+1)\frac{\pi}{2}$$

$$\therefore x = (2m+1)\frac{\cancel{\pi}}{2} \times \frac{\lambda}{2\cancel{\pi}}$$

$$x = (2m+1)\frac{\lambda}{4}$$

$$= \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$$

Thus antinodes are also separated by distance $\frac{\lambda}{2}$.



Hence, all the particles between successive nodes are inphase. during vibration the particles between successive nodes form a loop.
particles in adjacent loops are out of phase.
During vibration of string stationary loops are visible and hence the waves are called stationary or standing.

The length (l) of the string between fixed end points its integral multiple of $\lambda/2$ then only well defined stationary loops are observed.

* Normal Modes of Vibration of a String Stretched under a Tension.

Consider a string of length (l), fixed at both ends and stretched under a tension (T). when the String is excited by ~~the~~ striking transverse waves are created in the string which travel with a velocity $v = \sqrt{\frac{T}{m}}$.

if n is the frequency of vibration & λ the wavelength then, $v = n\lambda$.

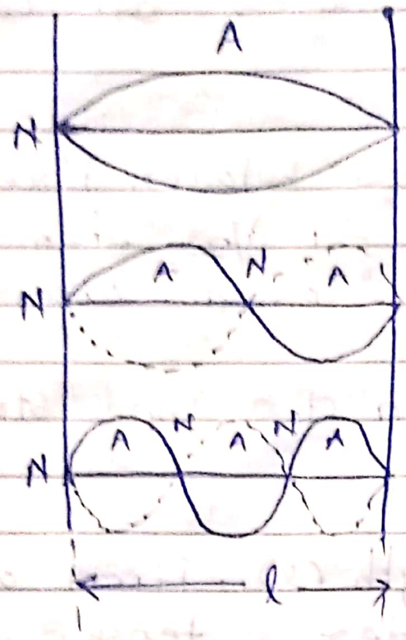
$\therefore$ Frequency of vibration of string

$$n = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}}.$$

If the length of the string is integral multiple of $\frac{\lambda}{2}$

ie. if $l = p \cdot \frac{\lambda}{2}$. where, $p = 1, 2, 3 \ldots$

Higher frequencies are called overtones. The quality of the sound is decided by the number of overtones present along with the fundamental and their relative amplituedes.



i) fundamental mode

ii) first overtone / second harmonic.

iii) second overtone / Third harmonic

i) Fundamental mode – In a vibrating string end points are always nodes (N). In fundamental mode one Antinode in between end - nodes.
ie the string vibrates forming a single loop
i) Therefore the wavelength (λ) corresponding to the fundamental mode is given by

$$l = \frac{\lambda}{2}$$

$\therefore$ Frequency of fundamental mode $n = \frac{V}{\lambda}$

$$= \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

Scanned with OKEN Scanner

ii) First overtone (or second harmonic)
one node and two antinodes in between end nodes
ie. the string vibrates forming two loops.
Therefore, wavelength (λ_1) corresponding to first overtone is given by $\lambda_1 = l$.

$\therefore$ Frequency of I^{st} overtone

$$n_1 = \frac{V}{\lambda_1} = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2n.$$

$$\boxed{n = \frac{1}{2l}\sqrt{\frac{T}{m}}}$$

iii) Second overtone (or third harmonic) :-
Here, two nodes and three antinodes in between end - nodes.
ie. The string vibrates forming 3 - loops so the wavelength corresponding to second overtone is given by,

$$l = 3 \cdot \frac{\lambda_2}{2} \quad \text{or} \quad \lambda_2 = \frac{2l}{3}.$$

$\therefore$ Frequency of second overtone is given by,

$$n_2 = \frac{V}{\lambda_2} = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n \quad \text{etc.}$$

Thus in general the frequency of $(p-1)^{th}$ overtone in which string vibrates forming p - loops is given by,

$$n_p = \frac{p}{2l}\sqrt{\frac{T}{m}} = p \cdot n \quad \text{where, } p = 1, 2, 3 \ldots$$

The frequencies of overtones are integral multiples of the fundamental frequency.